Intermediate Microeconomics Full Notes

Advanced Microeconomics (University of Nottingham)

Intermediate Microeconomics

Full Lecture Notes

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Contents

Ch. 1. The Market

I. Economic model: A simplified representation of reality

A. An example

– Rental apartment market in Shinchon: Object of our analysis

– Price of apt. in Shinchon: Endogenous variable

– Price of apt. in other areas: Exogenous variable

– Simplification: All (nearby) Apts are identical

B. We ask

– How the quantity and price are determined in a given allocation mechanism

– How to compare the allocations resulting from different allocation mechanisms

II. Two principles of economics

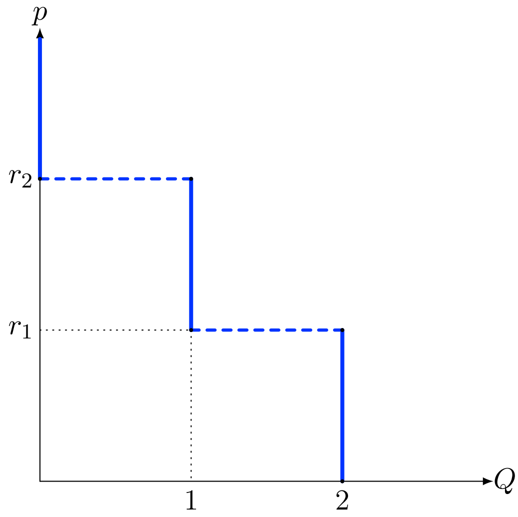
– Optimization principle: Each economic agent maximizes its objective (e.g. utility, profit, etc.)

– Equilibrium principle: Economic agents' actions must be consistent with each other

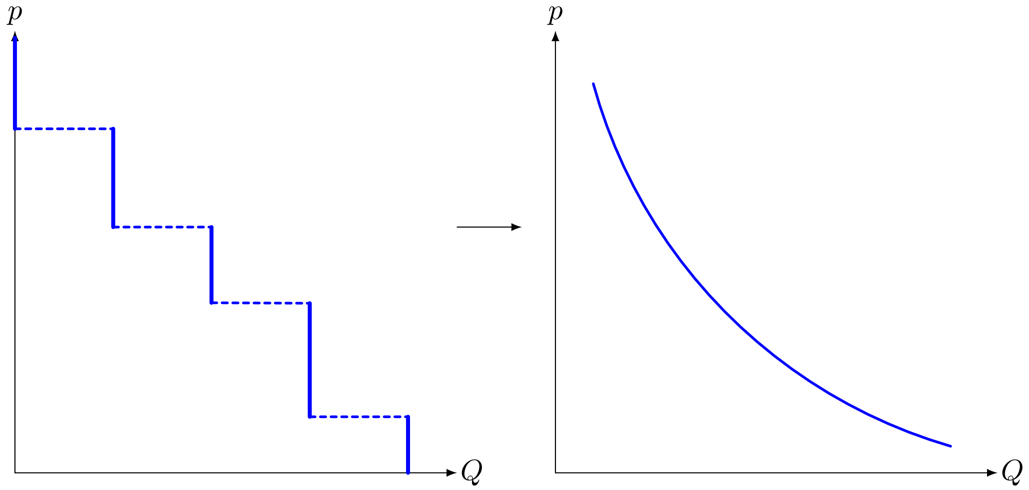
III. Competitive market

A. Demand

– Tow consumers with a single-unit demand whose WTP's are equal to r_1 and r_2 ($r_1 < r_2$)



– Many people



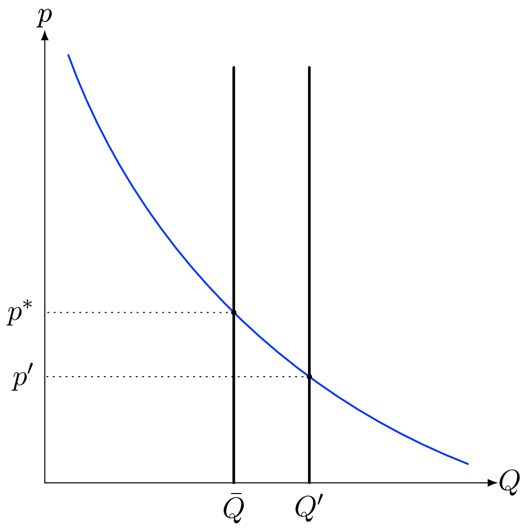
4 consumers $\quad\quad$ ∞ consumers

B. Supply

- Many competitive suppliers
- Fixed at $\bar{Q}$ in the short-run

C. Equilibrium

- Demand must equal supply



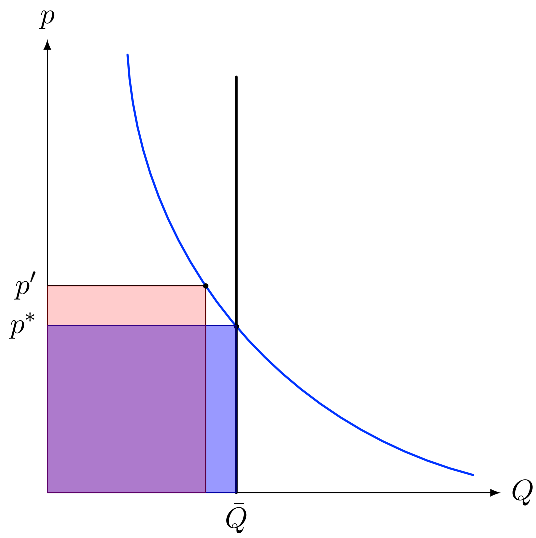
$\rightarrow$ Eq. price (p^*) and eq. quantity ($\bar{Q}$)

D. Comparative statics: Concerns how endogenous variables change as exogenous variables change

- $\begin{cases} \text{Comparative: Compare two eq'a} \\ \text{Statistics: Only look at eq'a, but not the adjustment process} \end{cases}$
- For instance, if there is exogenous increase in supply, $\bar{Q} \rightarrow Q'$, then $p^* \rightarrow p'$

III. Other allocation mechanisms

A. Monopoly



B. Rent control: Price ceiling at $\bar{p} < p^*$ $\rightarrow$ Excess demand $\rightarrow$ Rationing (or lottery)

IV. Pareto efficiency: Criterion to compare different economic allocations

A. One allocation is a *Pareto improvement* over the other if the former makes some people better off without making anyone else worse off, compared to the latter.

B. An allocation is called *Pareto efficient*(PE) if there is no Pareto improvement.

Otherwise, the allocation is called *Pareto inefficient*

C. Example: Rent control is not PE

- Suppose that there are 2 consumers, A and B, who value an apt at r_A and $r_B > r_A$.
- As a result of pricing ceiling and rationing, A gets an apt and B does not

– This is not Pareto efficient since there is Pareto improvement: Let A sell his/her apt. to B at the price of $p \in (r_A, r_B)$

	Before	After
Landlord	$\bar{p}$	$\bar{p}$
A	$r_A - \bar{p}$	$p - \bar{p}(> r_A - \bar{p})$
B	0	$r_B - p(> 0)$

$\rightarrow$ A and B are better off while no one is worse off

D. An allocation in the competitive market equilibrium is PE

Ch. 2. Budget Constraint

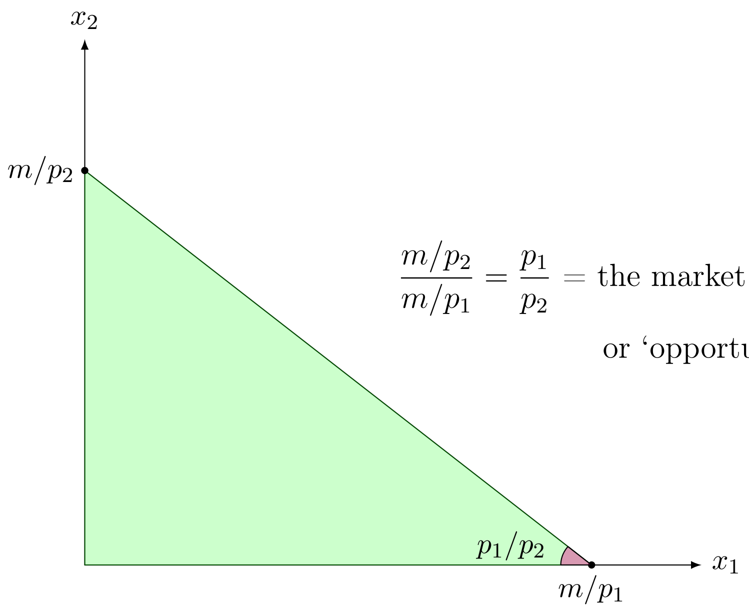
– Consumer's problem: Choose the 'best' bundle of goods that one 'can afford'

– Consider a consumer in an economy where there are 2 goods

– (x_1, x_2) : A bundle of two goods: Endogenous variable

– (p_1, p_2): Prices; m: Consumer's income: Exogenous variable

I. Budget set: Set of all *affordable* bundles $\rightarrow p_1x_1 + p_2x_2 \leq m$



$\frac{m/p_2}{m/p_1} = \frac{p_1}{p_2}$ = the market exchange rate b/w the two goods

or 'opportunity cost' of good 1 in terms of good 2

II. Changes in budget set

– See how budget set changes as exogenous variables change

A. Increase in income: $m < m'$

B. Increase in the price of one good: $p_1 < p_1'$

C. Proportional increase in all prices and income: $(p_1, p_2, m) \to (tp_1, tp_2, tm)$

※ Numeraire: Let $t = \frac{1}{p_1} \to \; x_1 + \frac{p_2}{p_1}x_2 = \frac{m}{p_1}$ that is, the price of good 1 is 1

III. Application: Tax and subsidy

A. Quantity tax: Tax levied on each unit of, say, good 1 bought

– Given tax rate t, $p_1' = p_1 + t$

B. Value tax: Tax levied on each dollar spent on good 1

– Given tax rate τ, $p_1' = p_1 + \tau p_1 = (1+\tau)p_1$

C. Subsidy: Negative tax

Example. $s =$ Quantity subsidy for the consumption of good 1 exceeding $\bar{x}_1$

Ch. 3. Preferences

I. Preference: Relationship (or rankings) between consumption bundles

A. Three modes of preference: Given two Bundles, $x = (x_1, x_2)$ and $y = (y_1, y_2)$

1. $x \succ y$: x 'is (strictly) preferred to' y
2. $x \sim y$: x 'is indifferent to' y
3. $x \succeq y$: x 'is weakly preferred to' y

   Example. $(x_1, x_2) \succeq (y_1, y_2)$ if $x_1 + x_2 \geq y_1 + y_2$

B. The relationships between three modes of preference

1. $x \succeq y \Leftrightarrow x \succ y$ or $x \sim y$
2. $x \sim y \Leftrightarrow x \succeq y$ and $y \succeq x$
3. $x \succ y \Leftrightarrow x \succeq y$ but not $y \succeq x$

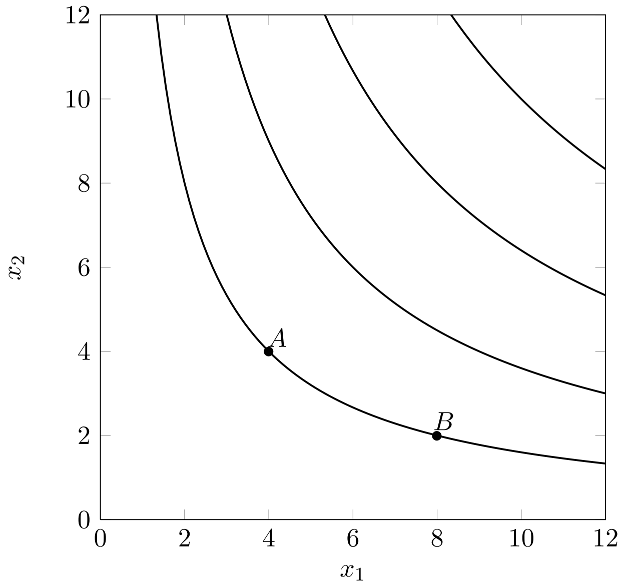
C. Properties of preference $\succeq$

1. $x \succeq y$ or $y \succeq x$
2. Reflexive: Given any x, $x \succeq x$
3. Transitive: Given x, y, and z, if $x \succeq y$ and $y \succeq z$, then $x \succeq z$

   Example. Does the preference in the above example satisfy all 3 properties?

   ※ If $\succeq$ is transitive, then $\succ$ and $\sim$ are also transitive: For instance, if $x \sim y$ and $y \sim z$, then $x \sim z$

D. Indifference curves: Set of bundles which are indifferent to one another



※ Two different indifferent curves cannot intersect with each other

※ Upper contour set: Set of bundles weakly preferred to a given bundle x

II. Well-behaved preference

A. Monotonicity: 'More is better'

– Preference is monotonic if $x \succeq y$ for any x and y satisfying $x_1 \geq y_1$, $x_2 \geq y_2$

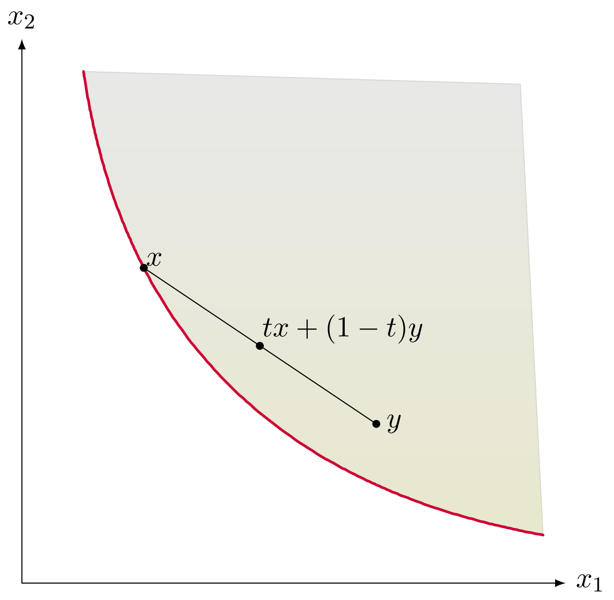
– Preference is *strictly* monotonic if $x \succ y$ for any x and y satisfying $x_1 \geq y_1$, $x_2 \geq y_2$, and $x \neq y$

Example. Monotonicity is violated by the satiated preference:

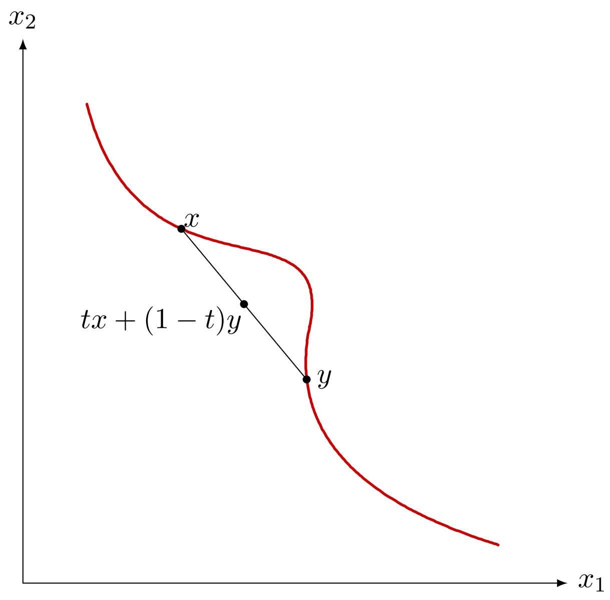
B. Convexity: 'Moderates are better than extremes'

– Preference is convex if for any x and y with $y \succeq x$, we have

$$tx + (1-t)y \succeq x \text{ for all } t \in [0,1]$$



Convex Preference



Non-convex Preference

→ Convex preference is equivalent to the convex upper contour set

– Preference is *strictly* convex if for any x and y with $y \succeq x$, we have

$$tx + (1-t)y \succ x \text{ for all } t \in (0, 1)$$

III. Examples

A. Perfect substitutes: Consumer likes two goods equally so only the total number of goods matters → 2 goods are perfectly substitutable

Example. Blue and Red pencil

B. Perfect complement: One good is useless without the other → It is not possible to substitute one good for the other

Example. Right and Left shoe

C. Bads: Less of a 'bad' is better

Example. Labor and Food

※ This preference violates the monotonicity but there is an easy fix: Let 'Leisure = 24 hours − Labor' and consider two goods, Leisure and Food.

IV. Marginal rate of substitution (MRS): MRS at a given bundle x is the marginal exchange rate between two goods to make the consumer indifferent to x.

$\rightarrow (x_1, x_2) \sim (x_1 - \Delta x_1, x_2 + \Delta x_2)$

$\rightarrow$ MRS at $x = \lim_{\Delta x_1 \to 0} \frac{\Delta x_2}{\Delta x_1} =$ slope of indifference curve at x

$\rightarrow$ MRS decreases as the amount of good 1 increases

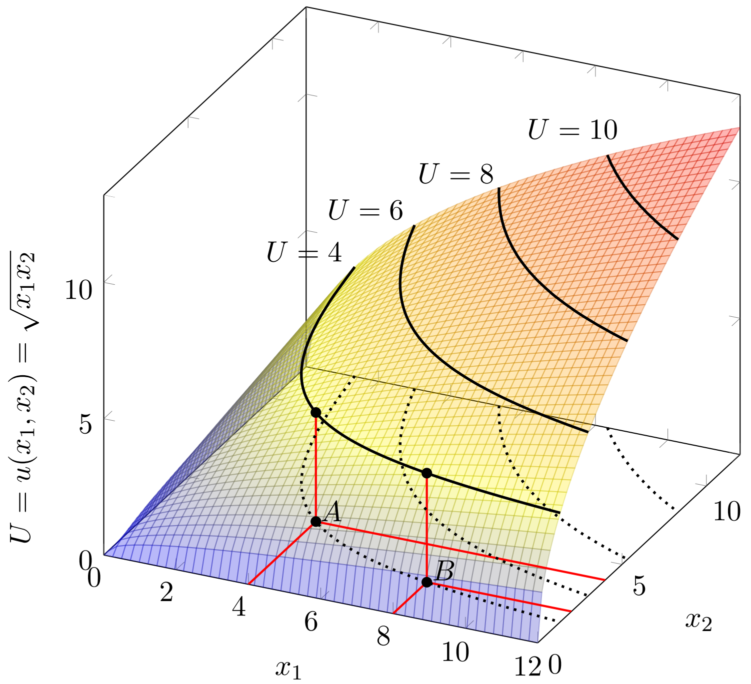
Ch. 4. Utility

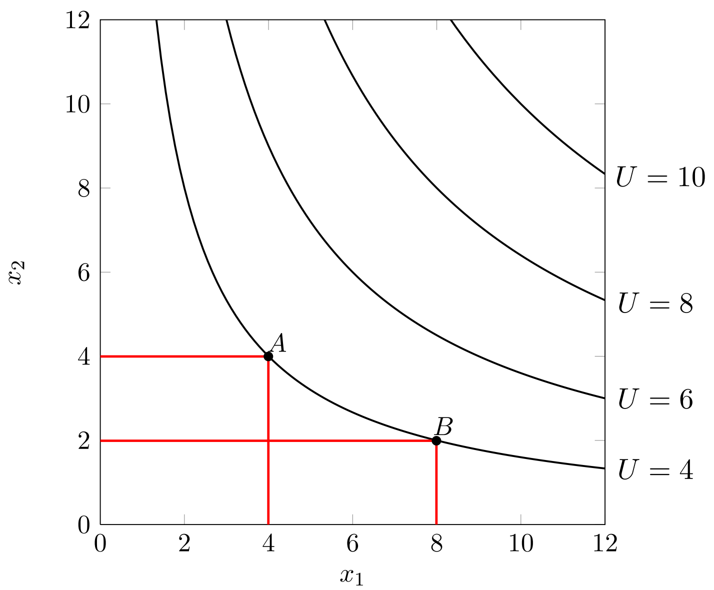
I. Utility function: An assignment of real number $u(x) \in \mathbb{R}$ to each bundle$x$

A. We say that u *represents* $\succ$ if the following holds:

$$x \succ y \text{ if and only if } u(x) > u(y)$$

– An indifference curve is the set of bundles that give the same level of utility:





B. Ordinal utility

– Only ordinal ranking matters while absolute level does *not* matter

Example. Three bundles x, y, and z, and $x \succ y \succ z \to$ Any $u(\cdot)$ satisfying $u(x) > u(y) > u(z)$ is good for representing $\succ$

– There are many utility functions representing the same preference

C. Utility function is *unique up to monotone transformation*

– For any increasing function $f : \mathbb{R} \to \mathbb{R}$, a utility function $v(x) \equiv f(u(x))$ represents the same preference as $u(x)$ since

$$x \succ y \Leftrightarrow u(x) > u(y) \Leftrightarrow v(x) = f(u(x)) > f(u(y)) = v(y)$$

D. Properties of utility function

– A utility function representing a *monotonic* preference must be increasing in x_1 and x_2

– A utility function representing a *convex* preference must satisfy: For any two bundles x and y,

$$u(tx + (1-t)y) \geq \min\{u(x), u(y)\} \text{ for all } t \in [0, 1]$$

II. Examples

A. Perfect substitutes

1. Red & blue pencils

   $\to u(x) = x_1 + x_2$ or $v(x) = (x_1 + x_2)^2$ ($\because v(x) = f(u(x))$, where $f(u) = u^2$)

2. One & five dollar bills

   $\to u(x) = x_1 + 5x_2$

3. In general, $u(x) = ax_1 + bx_2$

   $\to$ Substitution rate: $u(x_1 - \Delta x_1, x_2 + \Delta x_2) = u(x_1, x_2) \to \frac{\Delta x_2}{\Delta x_1} = \frac{a}{b}$

B. Perfect complements

1. Left & right shoes

   $$\to u(x) = \begin{cases} x_1 \text{ if } x_2 \geq x_1 \\ x_2 \text{ if } x_1 \geq x_2 \end{cases} \quad \text{or } u(x) = \min\{x_1, x_2\}$$

2. 1 spoon of coffee & 2 spoons of cream

   $$\to u(x) = \begin{cases} x_1 \text{ if } x_1 \leq \frac{x_2}{2} \\ \frac{x_2}{2} \text{ if } x_1 \geq \frac{x_2}{2} \end{cases} \quad \text{or } u(x) = \min\{x_1, \tfrac{x_2}{2}\} \text{ or } u(x) = \min\{2x_1, x_2\}$$

3. In general, $u(x) = \min\{ax_1, bx_2\}$, where $a, b > 0$

C. Cobb-Douglas: $u(x) = x_1^c x_2^d$, where $c, d > 0$

$\rightarrow v(x_1, x_2) = (x_1^c c_2^d)^{\frac{1}{c+d}} = x_1^{\frac{c}{c+d}} x_2^{\frac{d}{c+d}} = x_1^a x_2^{1-a}$, where $a \equiv \frac{c}{c+d}$

III. Marginal utility (MU) and marginal rate of substitution (MRS)

A. Marginal utility: The rate of the change in utility due to a marginal increase in one good *only*

– Marginal utility of good 1: $(x_1, x_2) \rightarrow (x_1 + \Delta x_1, x_2)$

$$MU_1 = \lim_{\Delta x_1 \to 0} \frac{\Delta U_1}{\Delta x_1} = \lim_{\Delta x_1 \to 0} \frac{u(x_1 + \Delta x_1, x_2) - u(x_1, x_2)}{\Delta x_1} \; (\rightarrow \Delta U_1 = MU_1 \times \Delta x_1)$$

– Analogously,

$$MU_2 = \lim_{\Delta x_2 \to 0} \frac{\Delta U_2}{\Delta x_2} = \lim_{\Delta x_2 \to 0} \frac{u(x_1, x_2 + \Delta x_2) - u(x_1, x_2)}{\Delta x_2} \; (\rightarrow \Delta U_2 = MU_2 \times \Delta x_2)$$

– Mathematically, $MU_i = \frac{\partial u}{\partial x_i}$, that is the partial differentiation of utility function u

B. MRS $\equiv \lim_{\Delta x_1 \to 0} \frac{\Delta x_2}{\Delta x_1}$ for which $u(x_1, x_2) = u(x_1 - \Delta x_1, x_2 + \Delta x_2)$

$$\begin{aligned}
\rightarrow 0 &= u(x_1 - \Delta x_1, x_2 + \Delta x_2) - u(x_1, x_2) \\
&= [u(x_1 - \Delta x_1, x_2 + \Delta x_2) - u(x_1, x_2 + \Delta x_2)] + [u(x_1, x_2 + \Delta x_2) - u(x_1, x_2)] \\
&= -[u(x_1, x_2 + \Delta x_2) - u(x_1 - \Delta x_1, x_2 + \Delta x_2)] + [u(x_1, x_2 + \Delta x_2) - u(x_1, x_2)] \\
&= -\Delta U_1 + \Delta U_2 = -MU_1 \Delta x_1 + MU_2 \Delta x_2
\end{aligned}$$

$$\rightarrow \text{MRS} = \frac{\Delta x_2}{\Delta x_1} = \frac{MU_1}{MU_2} = \frac{\partial u / \partial x_1}{\partial u / \partial x_2}$$

Example. $u(x) = x_1^a x_2^{1-a}$

$$\rightarrow \text{MRS} = \frac{MU_1}{MU_2} = \frac{a x_1^{a-1} x_2^{1-a}}{x_1^a (1-a) x_2^{-a}} = \frac{a x_2}{(1-a) x_1}$$

C. MRS is invariant with respect to the monotone transformation: Let $v(x) \equiv f(u(x))$ and then

$$\frac{\partial v / \partial x_1}{\partial v / \partial x_2} = \frac{f'(u) \cdot (\partial u / \partial x_1)}{f'(u) \cdot (\partial u / \partial x_2)} = \frac{\partial u / \partial x_1}{\partial u / \partial x_2}.$$

Example. An easier way to get MRS for the Cobb-Douglas utility function

$$u(x) = x_1^a x_2^{1-a} \rightarrow v(x) = a \ln x_1 + (1-a) \ln x_2$$

So, $MRS = \frac{MU_1}{MU_2} = \frac{a/x_1}{(1-a)/x_2} = \frac{a x_2}{(1-a) x_1}$

※ An alternative method for deriving MRS: Implicit function method

– Describe the indifference curve for a given utility level $\bar{u}$ by an implicit function $x_2(x_1)$ satisfying

$$u(x_1, x_2(x_1)) = \bar{u}$$

– Differentiate both sides with x_1 to obtain

$$\frac{\partial u(x_1, x_2)}{\partial x_1} + \frac{\partial u(x_1, x_2)}{\partial x_2}\frac{\partial x_2(x_1)}{\partial x_1} = 0,$$

which yields

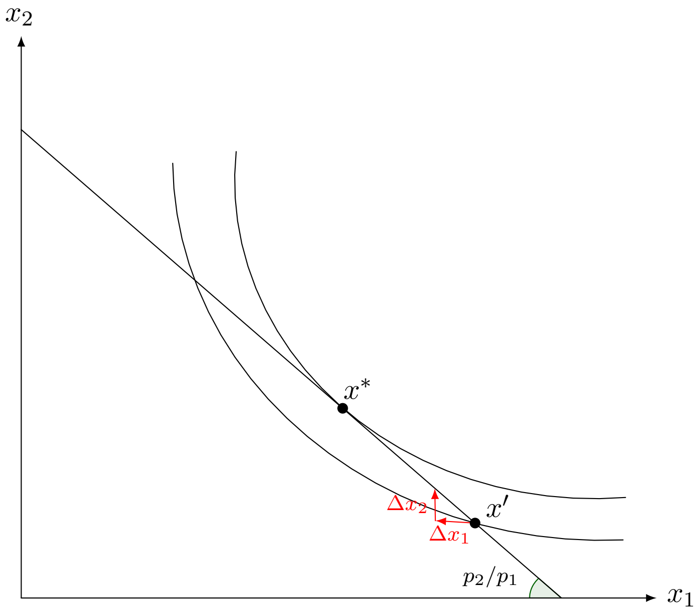
$$MRS = \left|\frac{\partial x_2(x_1)}{\partial x_1}\right| = \frac{\partial u(x_1, x_2)/\partial x_1}{\partial u(x_1, x_2)/\partial x_2}$$

Ch. 5. Choice

– Consumer's problem:

$$\text{Maximize } u(x_1, x_2) \quad \text{subject to} \quad p_1x_1 + p_2x_2 \le m$$

I. Tangent solution: Smooth and convex preference



x^* is optimal: $\boxed{\dfrac{MU_1}{MU_2} = MRS = \dfrac{p_1}{p_2}}$

– x' is not optimal: $\frac{MU_1}{MU_2} = MRS < \frac{p_1}{p_2} = \frac{\Delta x_2}{\Delta x_1}$ or $MU_1\Delta x_1 < MU_2\Delta x_2 \to$ Better off with exchanging good 1 for good 2

Example. Cobb-Douglas utility function

$$\left.\begin{array}{l} MRS = \frac{a}{1-a}\frac{x_2}{x_1} = \frac{p_1}{p_2} \\ p_1x_1 + p_2x_2 = m \end{array}\right\} \to (x_1^*, x_2^*) = \left(\frac{am}{p_1}, \frac{(1-a)m}{p_2}\right)$$

II. Non-tangent solution

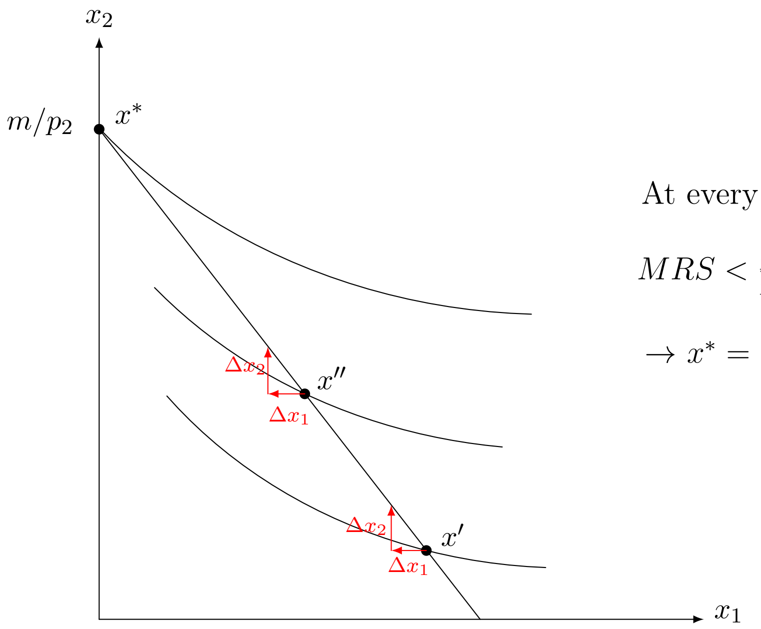
A. Kinked demand

Example. Perfect complement: $u(x_1, x_2) = \min\{x_1, x_2\}$

$$\left.\begin{array}{l} x_1 = x_2 \\ p_1 x_1 + p_2 x_2 = m \end{array}\right\} \to x_1^* = x_2^* = \frac{m}{p_1 + p_2}$$

B. Boundary optimum

1. No tangency:



At every bundle on the budget line,

$MRS < \frac{p_1}{p_2}$ or $\Delta x_1 \cdot MU_1 < \Delta x_2 \cdot MU_2$

$\to x^* = (0, m/p_2)$

Example. $u(x_1, x_2) = x_1 + \ln x_2$, $(p_1, p_2, m) = (4, 1, 3)$

$$MRS = x_2 < \frac{p_1}{p_2} = 4 \;\to\; \therefore x^* = (0, 3)$$

2. Non-convex preference: Beware of 'wrong' tangency

Example. $u(x) = x_1^2 + x_2^2$

3. Perfect substitutes:

$$\rightarrow (x_1^*, x_2^*) = \begin{cases} (m/p_1, 0) & \text{if } p_1 < p_2 \\ \text{any bundle on the budget line} & \text{if } p_1 = p_2 \\ (0, m/p_2) & \text{if } p_1 > p_2 \end{cases}$$

C. Application: Quantity vs. income tax

$$\begin{cases} \text{Quantity tax}: (p_1 + t)x_1 + p_2 x_2 = m \xrightarrow{\text{Utility max.}} (x_1^*, x_2^*) \text{ satisfying } p_1 x_1^* + p_2 x_2^* = m - t x_1^* \\ \text{Income tax}: p_1 x_1 + p_2 x_2 = m - R \xrightarrow{\text{Set } R = t x_1^*} p_1 x_1 + p_2 x_2 = m - t x_1^* \end{cases}$$

$\rightarrow$ Income tax that raises the same revenue as quantity tax is better for consumers

Appendix: Lagrangian Method

I. General treatment (cookbook procedure)

- Let f and $g_j, j = 1, \cdots, J$ be functions mapping from $\mathbb{R}^n$ and $\mathbb{R}$.
- Consider the constrained maximization problem as follows:

$$\max_{x=(x_1,\cdots x_n)} f(x) \quad \text{subject to } g_j(x) \geq 0, j = 1, \cdots, J. \tag{A.1}$$

So there are J constraints, each of which is represented by a function g_j.

- Set up the *Lagrangian function* as follows

$$L(x, \lambda) = f(x) + \sum_{j=1}^{J} \lambda_j g_j(x).$$

We call $\lambda_j, j = 1, \cdots, J$ *Lagrangian multipliers.*

- Find a vector (x^*, λ^*) that solves the following equations:

$$\begin{aligned} &\tfrac{\partial L(x^*,\lambda^*)}{\partial x_i} = 0 \text{ for all } i = 1, \cdots, n \\ &\lambda_j^* g_j(x^*) = 0 \text{ and } \lambda_j^* \geq 0 \text{ for all } j = 1, \cdots, J. \end{aligned} \tag{A.2}$$

- Kuhn-Tucker theorem tells us that x^* is the *solution of the original maximization problem* given in (A.1), provided that some concavity conditions hold for f and $g_j, j = 1, \cdots J$. (For details, refer to any textbook in the mathematical economics.)

II. Application: Utility maximization problem

- Set up the utility maximization problem as follows:

$$\max_{x=(x_1,x_2)} u(x)$$

subject to

$$m - p_1 x_1 - p_2 x_2 \geq 0$$

$$x_1 \geq 0 \text{ and } x_2 \geq 0.$$

- The Lagrangian function corresponding to this problem can be written as

$$L(x, \lambda) = u(x) + \lambda_3(m - p_1 x_1 - p_2 x_2) + \lambda_1 x_1 + \lambda_2 x_2$$

A. Case of interior solution: Cobb-Douglas utility, $u(x) = a \ln x_1 + (1 - a) \ln x_2$, $a \in (0, 1)$

– The Lagrangian function becomes

$$L(x,\lambda) = a\ln x_1 + (1-a)\ln x_2 + \lambda_3(m - p_1x_1 - p_2x_2) + \lambda_1x_1 + \lambda_2x_2$$

– Then, the equations in (A.2) can be written as

$$\frac{\partial L(x^*,\lambda^*)}{\partial x_1} = \frac{a}{x_1^*} - \lambda_3^*p_1 + \lambda_1^* = 0 \tag{A.3}$$

$$\frac{\partial L(x^*,\lambda^*)}{\partial x_2} = \frac{1-a}{x_2^*} - \lambda_3^*p_2 + \lambda_2^* = 0 \tag{A.4}$$

$$\lambda_3^*(m - p_1x_1^* - p_2x_2^*) = 0 \text{ and } \lambda_3^* \geq 0 \tag{A.5}$$

$$\lambda_1^*x_1^* = 0 \text{ and } \lambda_1^* \geq 0 \tag{A.6}$$

$$\lambda_2^*x_2^* = 0 \text{ and } \lambda_2^* \geq 0 \tag{A.7}$$

1) One can easily see that $x_1^* > 0$ and $x_2^* > 0$ so $\lambda_1^* = \lambda_2^* = 0$ by (A.6) and (A.7).
2) Plugging $\lambda_1^* = \lambda_2^* = 0$ into (A.3), we can see $\lambda_3^* = \frac{a}{p_1x_1^*} > 0$, which by (A.5) implies

$$m - p_1x_1^* - p_2x_2^* = 0. \tag{A.8}$$

3) Combining (A.3) and (A.4) with $\lambda_1^* = \lambda_2^* = 0$, we are able to obtain

$$\frac{ax_2^*}{(1-a)x_1^*} = \frac{p_1}{p_2} \tag{A.9}$$

4) Combining (A.8) and (A.9) yields the solution for $(x_1^*, x_2^*) = (\frac{am}{p_1}, \frac{(1-a)m}{p_2})$, which we have seen in the class.

B. Case of boundary solution: Quasi-linear utility, $u(x) = x_1 + \ln x_2$.

– Let $(p_1, p_2, m) = (4, 1, 3)$

– The Lagrangian becomes

$$L(x,\lambda) = x_1 + \ln x_2 + \lambda_3(3 - 4x_1 - x_2) + \lambda_1x_1 + \lambda_2x_2$$

– Then, the equations in (A.2) can be written as

$$\frac{\partial L(x^*,\lambda^*)}{\partial x_1} = 1 - 4\lambda_3^* + \lambda_1^* = 0 \tag{A.10}$$

$$\frac{\partial L(x^*,\lambda^*)}{\partial x_2} = \frac{1}{x_2^*} - \lambda_3^* + \lambda_2^* = 0 \tag{A.11}$$

$$\lambda_3^*(3 - 4x_1^* - x_2^*) = 0 \text{ and } \lambda_3^* \geq 0 \tag{A.12}$$

$$\lambda_1^*x_1^* = 0 \text{ and } \lambda_1^* \geq 0 \tag{A.13}$$

$$\lambda_2^*x_2^* = 0 \text{ and } \lambda_2^* \geq 0 \tag{A.14}$$

1) One can easily see that $x_2^* > 0$ so $\lambda_2^* = 0$ by (A.14).

2) Plugging $\lambda_2^* = 0$ into (A.11), we can see $\lambda_3^* = \frac{1}{x_2^*} > 0$, which by (A.12) implies

$$3 - 4x_1^* - x_2^* = 0. \tag{A.15}$$

3) By (A.10),

$$\lambda_1^* = 4\lambda_3^* - 1 = \frac{4}{x_2^*} - 1 > 0 \tag{A.16}$$

since $x_2^* \leq 3$ due to (A.15).

4) Now, (A.16) and (A.13) imply $x_1^* = 0$, which in turn implies $x_2^* = 3$ by (A.15).

Ch. 6. Demand

– We studied how the consumer maximizes utility given p and m

$\rightarrow$ Demand function: $x(p,m) = (x_1(p,m), x_2(p,m))$

– We ask here how $x(p,m)$ changes with p and m?

I. Comparative statics: Changes in income

A. Normal or inferior good: $\frac{\partial x_i(p,m)}{\partial m} > 0$ or < 0

B. Income offer curves and Engel curves

C. Homothetic utility function

– For any two bundles x and y, and any number $\alpha > 0$,

$$u(x) > u(y) \Leftrightarrow u(\alpha x) > u(\alpha y).$$

– Perfect substitute and complements, and Cobb-Douglas are all homothetic.

– x^* is a utility maximizer subject to $p_1 x_1 + p_2 x_2 \leq m$ if and only tx^* is a utility maximizer subject to $p_1 x_1 + p_2 x_2 \leq tm$ for any $t > 0$.

– Income offer and Engel curves are straight lines

D. Quasi-linear utility function: $u(x_1, x_2) = x_1 + v(x_2)$, where v is a concave function, that is v' is decreasing.

– Define x_2^* to satisfy

$$MRS = \frac{1}{v'(x_2)} = \frac{p_1}{p_2} \tag{1}$$

– If m is large enough so that $m \geq p_2 x_2^*$, then the tangent condition (1) can be satisfied.
$\rightarrow$ The demand of good 2, x_2^*, does not depend on the income level

– If $m < p_2 x_2^*$, then the LHS of (1) is always greater than the RHS for any $x_2 \leq \frac{m}{p_2}$
$\rightarrow$ Boundary solution occurs at $x^* = (0, \frac{m}{p_2})$.

Example. Suppose that $u(x_1, x_2) = x_1 + \ln x_2$, $(p_1, p_2) = (4, 1)$. Draw the income offer and Engel curves.

II. Comparative statics: Changes in price

A. Ordinary or Giffen good: $\frac{\partial x_i(p,m)}{\partial p_i} < 0$ or >0

B. Why Giffen good?

$$p_1 \nearrow \longrightarrow \begin{cases} \text{Relatively more expensive good } 1 : x_1 \searrow \\ \text{Reduced real income} \begin{cases} \text{normal good: } x_1 \searrow \\ \text{inferior good: } x_1 \nearrow \end{cases} \end{cases}$$

So, a good must be inferior in order to be Giffen

C. Price offer curves and demand curves

D. Complements or substitutes: $\frac{\partial x_i(p,m)}{\partial p_j} < 0$ or > 0

Ch. 7. Revealed Preference

I. Revealed preference: Choice (observable) reveals preference (unobservable)

– Consumer's observed choice: $\begin{cases} (x_1, x_2) \text{ chosen under price } (p_1, p_2) \\ (y_1, y_2) \text{ chosen under price } (q_1, q_2) \end{cases}$

A. '*Directly revealed preferred*' (d.r.p.)

$$(x_1, x_2) \text{ is d.r.p. to } (y_1, y_2) \Leftrightarrow p_1x_1 + p_2x_2 \geq p_1y_1 + p_2y_2$$

B. '*Indirectly revealed preferred*' (i.r.p.)

$$(x_1, x_2) \text{ is i. r. p. to } (z_1, z_2)$$

C. '*Revealed preferred*' (r.p.) $=$ 'd.r.p. or i.r.p.'

II. Axioms of revealed preference:

A. Weak axiom of reveal preference (WARP):

– If (x_1, x_2) is d.r.p. to (y_1, y_2) with $(x_1, x_2) \neq (y_1, y_2),$ then (y_1, y_2) must not be d.r.p. to (x_1, x_2)
$\Leftrightarrow$ If $p_1x_1 + p_2x_2 \geq p_1y_1 + p_2y_2$ with $(x_1, x_2) \neq (y_1, y_2),$ then it must be that $q_1y_1 + q_2y_2 < q_1x_1 + q_2x_2$

B. How to check WARP

– Suppose that we have the following observations

Observation	Prices	Bundle
1	(2,1)	(2,1)
2	(1,2)	(1,2)
3	(2,2)	(1,1)

– Calculate the costs of bundles

Price \ Bundle	1	2	3
1	5	4^*	3^*
2	4^*	5	3^*
3	6	6	4

→ WARP is violated since bundle 1 is d.r.p. to bundle 2 under price 1 while bundle 2 is d.r.p. to bundle 1 under price 2.

C. Strong axiom of revealed preference (SARP) If (x_1, x_2) is r.p. to (y_1, y_2) with $(x_1, x_2) \neq (y_1, y_2)$, then (y_1, y_2) must not be r.p. to (x_1, x_2)

III. Index numbers and revealed preference

– Enables us to measure consumers' welfare without information about their actual preferences

– Consider the following observations $\begin{cases} \text{Base year} : (x_1^b, x_2^b) \text{ under } (p_1^b, p_2^b) \\ \text{Current year} : (x_1^t, x_2^t) \text{ under } (p_1^t, p_2^t) \end{cases}$

A. Quantity indices: Measure the change in "average consumptions"

– $I_q = \dfrac{\omega_1 x_1^t + \omega_2 x_2^t}{\omega_1 x_1^b + \omega_2 x_2^b}$, where ω_i is the weight for good $i = 1, 2$

– Using prices as weights, we obtain $\begin{cases} \text{Passhe quantity index } (P_q) \text{ if } (\omega_1, \omega_2) = (p_1^t, p_2^t) \\ \text{Laspeyres quantity index } (L_q) \text{ if } (\omega_1, \omega_2) = (p_1^b, p_2^b) \end{cases}$

B. Quantity indices and consumer welfare

– $P_q = \dfrac{p_1^t x_1^t + p_2^t x_2^t}{p_1^t x_1^b + p_2^t x_2^b} > 1$: Consumer must be better off at the current year

– $L_q = \dfrac{p_1^b x_1^t + p_2^b x_2^t}{p_1^b x_1^b + p_2^b x_2^b} < 1$: Consumer must be worse off at the current year

C. Price indices: Measure the change in "cost of living"

- $I_p = \dfrac{x_1 p_1^t + x_2 p_2^t}{x_1 p_1^b + x_2 p_2^b}$, where (x_1, x_2) is a fixed bundle
- Depending on what bundle to use, we obtain $\begin{cases} \text{Passhe price index } (P_p) : (x_1, x_2) = (x_1^t, x_2^t) \\ \text{Laspeyres price index } (L_p) : (x_1, x_2) = (x_1^b, x_2^b) \end{cases}$
- Laspeyres price index is also known as "consumer price index (CPI)": This has problem of overestimating the change in cost of living

Ch. 8. Slutsky Equation

– Change of price of one good: $p_1 \to p'_1$ with $p'_1 > p_1$

$$\to \begin{cases} \text{Change in relative price } (p_1/p_2) \to \text{Substitution effect} \\ \text{Change in real income} \to \text{Income effect} \end{cases}$$

I. Substitution and income effects:

– Aim to decompose the change $\Delta x_1 = x_1(p'_1, p_2, m) - x_1(p_1, p_2, m)$ into the changes due to the substitution and income effects.

– To obtain the change in demand due to substitution effect,

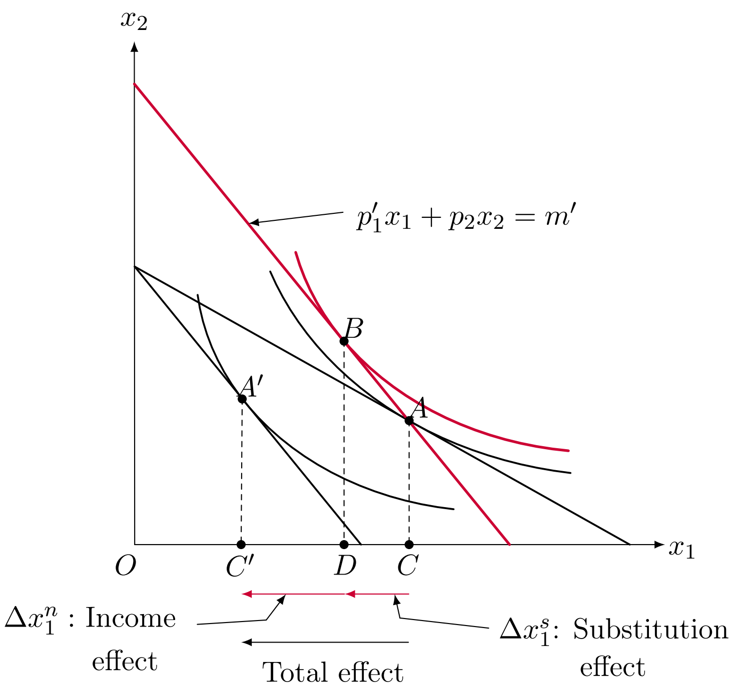
(1) *compensate* the consumer so that the original bundle is affordable under (p'_1, p_2)

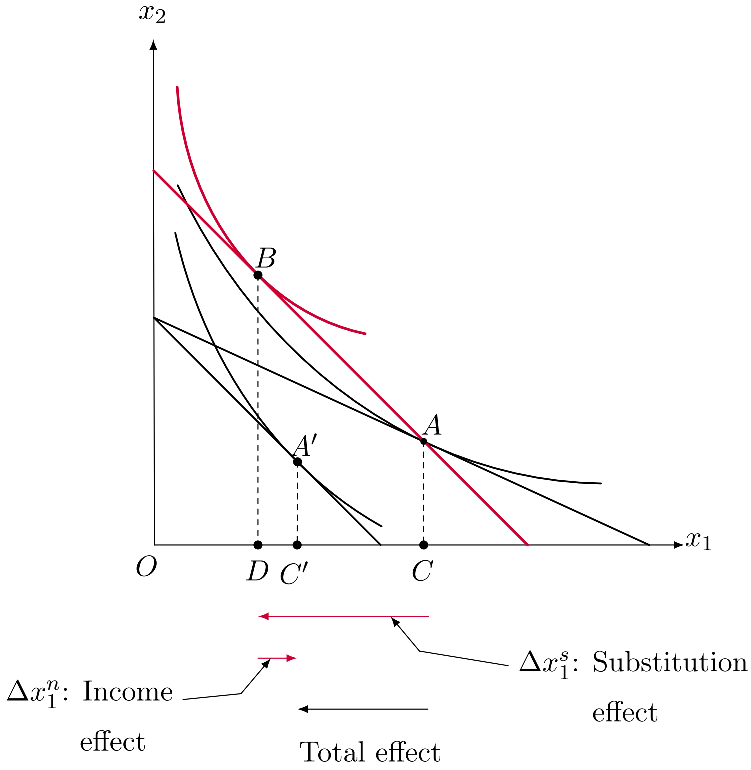
$$\to m' = p'_1 x_1(p_1, p_2, m) + p_2 x_2(p_1, p_2, m)$$

(2) ask what bundle he chooses under $(p'_1, p_2, m') \to x_1(p'_1, p_2, m')$

(3) decompose Δx_1 as follows:

$$\begin{aligned} \Delta x_1 &= x_1(p'_1, p_2, m) - x_1(p_1, p_2, m) \\ &= \underbrace{[x_1(p'_1, p_2, m') - x_1(p_1, p_2, m)]}_{\substack{\Delta x_1^s \text{ : Change due to} \\ \text{substitution effect}}} + \underbrace{[x_1(p'_1, p_2, m) - x_1(p'_1, p_2, m')]}_{\substack{\Delta x_1^n \text{ : Change due to} \\ \text{income effect}}} \end{aligned}$$





– Letting $\Delta p_1 \equiv p_1' - p_1$, $\begin{cases} \text{Substitution Effect} : \frac{\Delta x_1^s}{\Delta p_1} < 0 \\ \text{Income Effect} \begin{cases} \frac{\Delta x_1^n}{\Delta p_1} < 0 \text{ if good 1 is normal} \\ \frac{\Delta x_1^n}{\Delta p_1} > 0 \text{ if good 1 is inferior} \end{cases} \end{cases}$

$\rightarrow$ In case of inferior good, if the income effect dominates the substitution effect, then there arises a Giffen phenomenon.

Example. Cobb-Douglas with $a = 0.5,\ p_1 = 2,\ \&\ m = 16 \rightarrow p_1' = 4$

Remember $x_1(p, m) = \frac{am}{p_1}$ so

$$\begin{aligned}
&x_1(p_1, p_2, m) = 0.5 \times (16/2) = 4,\ x_1(p_1', p_2, m) = 0.5 \times (16/4) = 2 \\
&m' = m + (m' - m) = m + (p_1' - p_1)x_1(p_1, p_2, m) = 16 + (4 - 2)4 = 24 \\
&x_1(p_1', p_2, m') = 0.5 \times (24/4) = 3 \\
&\therefore \Delta x_1^s = x_1(p_1', p_2, m') - x_1(p_1, p_2, m) = 3 - 4 = -1 \\
&\quad \Delta x_1^n = x_1(p_1', p_2, m) - x_1(p_1', p_2, m') = 2 - 3 = -1
\end{aligned}$$

II. Slutsky equation

– Letting $\Delta m \equiv m' - m$, we have $\Delta m = m' - m = (p_1' - p_1)x_1(p, m) = \Delta p_1 x_1(p, m)$

A. For convenience, let $\Delta x_1^m \equiv -\Delta x_1^n$. Then,

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1^m}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1^m}{\Delta m} x_1(p, m)$$

B. Law of demands (restated): If the good is normal, then its demand must fall as the price rises

III. Application: Rebating tax on gasoline:

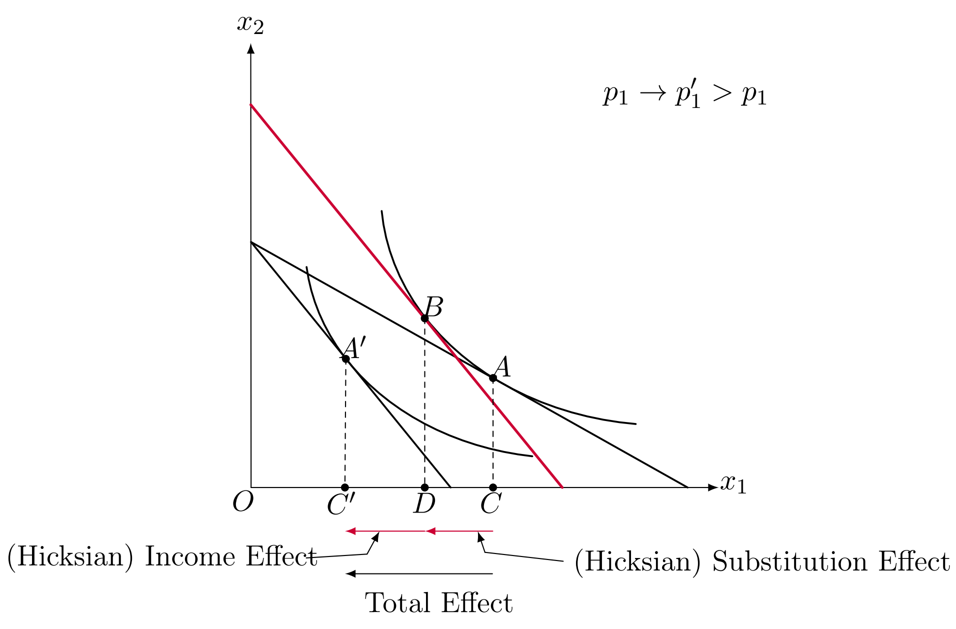
- $\begin{cases} x = \text{Consumption of gasolin} \\ y = \text{Expenditure (\$) on all other goods, whose price is normalized to 1} \end{cases}$
- $\begin{cases} t: \text{ Quantity tax on gasollin} \rightarrow p' = p + t \\ (x', y'): \text{ Choice after tax } t \text{ and rebate } R = tx' \end{cases}$
- With (x', y'), we have $(p+t)x' + y' = m + tx'$ or $px' + y' = m$

  $\rightarrow$ (x', y') must be on both budget lines

  $\rightarrow$ Optimal bundle before the tax must be located to the left of (x', y'), that is the gasoline consumption must have decreased after the tax-rebate policy

IV. Hicksian substitution effect

- Make the consumer be able to achieve the same (original) utility instead the same (original) bundle



Ch. 9. Buying and Selling

– Where does the consumer's income come from? $\rightarrow$ Endowment$= (\omega_1, \omega_2)$

I. Budget constraint

A. Budget line: $p_1x_1 + p_2x_2 = p_1\omega_1 + p_2\omega_2 \rightarrow$ Always passes through (ω_1, ω_2)

B. Change in price: $p_1 : (x_1, x_2) \rightarrow p_1' : (x_1', x_2')$ with $p_1' < p_1$

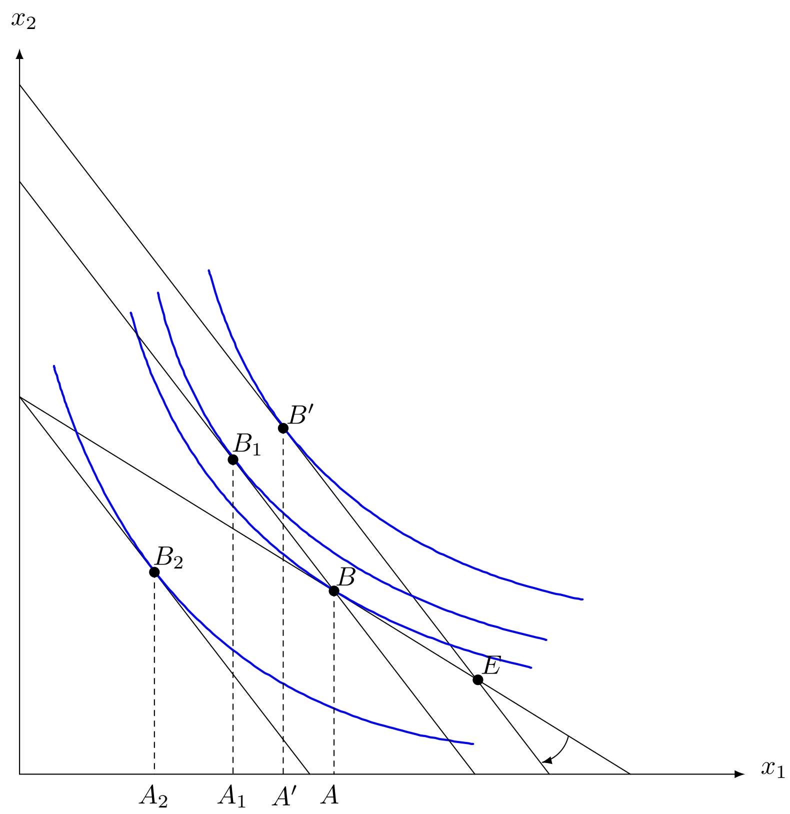
$$\rightarrow \text{Consumer welfare} \begin{cases} x_1 - \omega_1 > 0 \rightarrow x_1' - \omega_1 > 0 : \text{ Better off} \\ x_1 - \omega_1 < 0 \left\langle \begin{array}{l} x_1' - \omega_1 < 0 : \text{ Worse off} \\ x_1' - \omega_1 > 0 : \text{ ?} \end{array} \right. \end{cases}$$

II. Slutsky equation

– Suppose that p_1 increases to $p_1' > p_1$

A. Endowment income effect

– Suppose that the consumer chooses B under p_1 and B' under p_1'



– The change in consumption of good 1 from A to A' can be decomposed into

$$\begin{cases} A \to A_1: \text{ Substitution effect} \\ A_1 \to A_2: \text{ Ordinary income effect} \\ A_2 \to A': \textit{ Endowment} \text{ income effect} \end{cases}$$

– A good is Giffren if its demand decreases with its own price with income being *fixed*

B. Slutsky equation

– The original bundle $A = (x_1, x_2)$ would be affordable under (p_1', p_2) and compensation Δm if Δm satisfies

$$p_1'x_1 + p_2x_2 = p_1'\omega_1 + p_2\omega_2 + \Delta m,$$

from which Δm can be calculated as

$$\begin{array}{rrl} & p_1'x_1 + p_2x_2 = & p_1'\omega_1 + p_2\omega_2 + \Delta m \\ - & p_1'x_1 + p_2x_2 = & p_1'\omega_1 + p_2\omega_2 \\ \hline & (p_1' - p_1)x_1 = & (p_1' - p_1)\omega_1 + \Delta m \\ & \therefore \Delta m = & -\Delta p_1(\omega_1 - x_1) \end{array}$$

– Then, the Slutsky equation is given as

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1^m}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1^m}{\Delta m}\frac{\Delta m}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} + \frac{\Delta x_1^m}{\Delta m}(\omega_1 - x_1)$$

$$\to \begin{cases} \dfrac{\Delta x_1^s}{\Delta p_1}: \text{ Substitution effect} \\ \dfrac{\Delta x_1^m}{\Delta m}(-x_1): \text{ Ordinary income effect} \leftarrow \text{ decrease in real income by } -x_1\Delta p_1 \\ \dfrac{\Delta x_1^m}{\Delta m}\omega_1: \text{ Endowment income effect} \leftarrow \text{ increase in monetary income by } \omega_1\Delta p_1 \end{cases}$$

III. Application: labor supply

– $\begin{cases} C: \text{ Consumption good} \\ p: \text{ Price of consumption good} \\ \ell: \text{ Leisure time; } \overline{L}: \text{ endowment of time} \\ w: \text{ Wage = price of leisure} \\ M: \text{ Non-labor income} \\ \overline{C} \equiv M/P: \text{ Consumption available when being idle} \end{cases}$

– $U(C, \ell)$: Utility function, increasing in both C and ℓ

– $L = \overline{L} - \ell$, labor supply

A. Budget constraint and optimal labor supply

$$pC = M + wL \Leftrightarrow M = pC - wL = pC - w(\overline{L} - \ell) \Leftrightarrow pC + wl = M + w\overline{L} = \underbrace{p\overline{C} + w\overline{L}}_{\text{value of endowment}}$$

e.g.) Assume $U(C, l) = C^a \ell^{1-a}$, $0 < a < 1$, $M = 0$, and $\overline{L} = 16$, and derive the labor supply curve

B. Changes in wage: $w < w'$

- Note that the leisure is not Giffen since the increase in its price (or wage increase) makes income increase also
- Backward bending labor supply curve: Labor supply can be decreasing as wage increases

Ch. 10. Intertemporal Choice

– Another application of buy-and-selling model

– Choice problem involving saving and consuming over time

I. Setup

– A consumer who lives for 2 periods, period 1 (today) and period 2 (tomorrow)

– (c_1, c_2): Consumption plan, c_i = consumption in period i

– (m_1, m_2): Income stream, m_i = income in period i

– r: interest rate, that is saving \$1 today earns \$$(1+r)$ tomorrow

– Utility function: $U(c_1, c_2) = u(c_1) + \delta u(c_2)$, where $\delta < 1$ is discount rate, and u is concave (that is u' is decreasing)

II. Budget constraint

– $s \equiv m_1 - c_1$: saving(+) or borrowing(−) in period 1

– Budget equation: $c_2 = m_2 + (1+r)s = m_2 + (1+r)(m_1 - c_1)$, from which we obtain

$$\underbrace{c_1 + \frac{1}{1+r}c_2}_{\substack{\text{present value}\\ \text{of consumption}\\ \text{plan } (c_1, c_2)}} = \underbrace{m_1 + \frac{1}{1+r}m_2}_{\substack{\text{present value}\\ \text{of income}\\ \text{stream } (m_1, m_2)}}$$

※ Present value

– Present value (PV) of amount x in t periods from now $= \frac{x}{(1+r)^t}$

– PV of a job that will earn m_t in period $t = 1, \cdots, T$

$$= \frac{m_1}{1+r} + \frac{m_2}{(1+r)^2} + \cdots + \frac{m_T}{(1+r)^T} = \sum_{t=1}^{T} \frac{m_t}{(1+r)^t}$$

– PV of a consol that promises to pay \$$x$ per year for ever

$$= \sum_{t=1}^{\infty} \frac{x}{(1+r)^t} = \frac{\frac{x}{1+r}}{1 - \frac{1}{1+r}} = \frac{x}{r}$$

III. Choice

– Maximize $u(c_1) + \delta u(c_2)$ subject to $c_1 + \frac{1}{1+r}c_2 = m_1 + \frac{1}{1+r}m_2$

– Tangency condition:

$$\frac{u'(c_1)}{\delta u'(c_2)} = \frac{MU_1}{MU_2} = \text{slope of budget line} = \frac{1}{1/(1+r)} = 1 + r$$

or

$$\frac{u'(c_1)}{u'(c_2)} = \delta(1+r)$$

$$\rightarrow \begin{cases} c_1 = c_2 & \text{if } (1+r) = 1/\delta \\ c_1 > c_2 & \text{if } (1+r) < 1/\delta \\ c_1 < c_2 & \text{if } (1+r) > 1/\delta \end{cases}$$

Ch. 12. Uncertainty

– Study the consumer's decision making under uncertainty

– Applicable to the analysis of lottery, insurance, risky asset, and many other problems

I. Insurance problem

A. Contingent consumption: Suppose that there is a consumer who derives consumption from a financial asset that has uncertain value:

– Two states $\begin{cases} \text{bad state, with prob. } \pi \\ \text{good state, with prob. } 1-\pi \end{cases}$

– Value of asset $\begin{cases} m_b \text{ in bad state} \\ m_g \text{ in good state} \end{cases} \rightarrow$ Endowment

– Contingent consumption $\begin{cases} \text{consumption in bad state} \equiv c_b \\ \text{consumption in good state} \equiv c_g \end{cases}$

– Insurance $\begin{cases} K: \text{ Amount of insurance purchased} \\ \gamma: \text{ Premium per dollar of insurance} \end{cases}$

– Purchasing \$$K$ of insurance, the contingent consumption is given as

$$c_b = m_b + K - \gamma K \tag{2}$$

$$c_g = m_g - \gamma K \tag{3}$$

B. Budget constraint

– Obtaining $K = \frac{m_g - c_g}{\gamma}$ from (3) and substituting it in (2) yields

$$c_b = m_b + (1-\gamma)K = m_b + (1-\gamma)\frac{m_g - c_g}{\gamma}$$

$$\text{or} \quad c_b + \frac{(1-\gamma)}{\gamma}c_g = m_b + \frac{(1-\gamma)}{\gamma}m_g$$

II. Expected utility

A. Utility from the consumption plan (c_b, c_g):

$$\pi u(c_b) + (1-\pi)u(c_g)$$

$\rightarrow$ Expected utility: Utility of prize in each state is weighted by its probability

※ In general, if there are n states with state $i$occurring with probability π_i, then the expected utility is given as

$$\sum_{i=1}^{n} \pi_i u(c_i).$$

B. Why expected utility?

– Independence axiom

Example. Consider two assets as follows:

$$A_1 \begin{cases} \text{State 1} : \$1M \\ \text{State 2} : \$0.5M \\ \text{State 3} : \$x \end{cases} \qquad \text{or} \qquad A_2 \begin{cases} \text{State 1} : \$1.5M \\ \text{State 2} : \$0 \\ \text{State 3} : \$x \end{cases}$$

$\rightarrow$ Independence axiom requires that if A_1 is preferred to A_2 for *some* x, then it must be the case for *all other* x.

– According to Independence axiom, comparison between prizes in two states (State 1 and 2) should be independent of the prize (x) in any third state (State 3)

C. Expected utility is unique up to the affine transformation: Expected utility functions U and V represent the same preference if and only if

$$V = aU + b,\ a > 0$$

D. Attitude toward risk

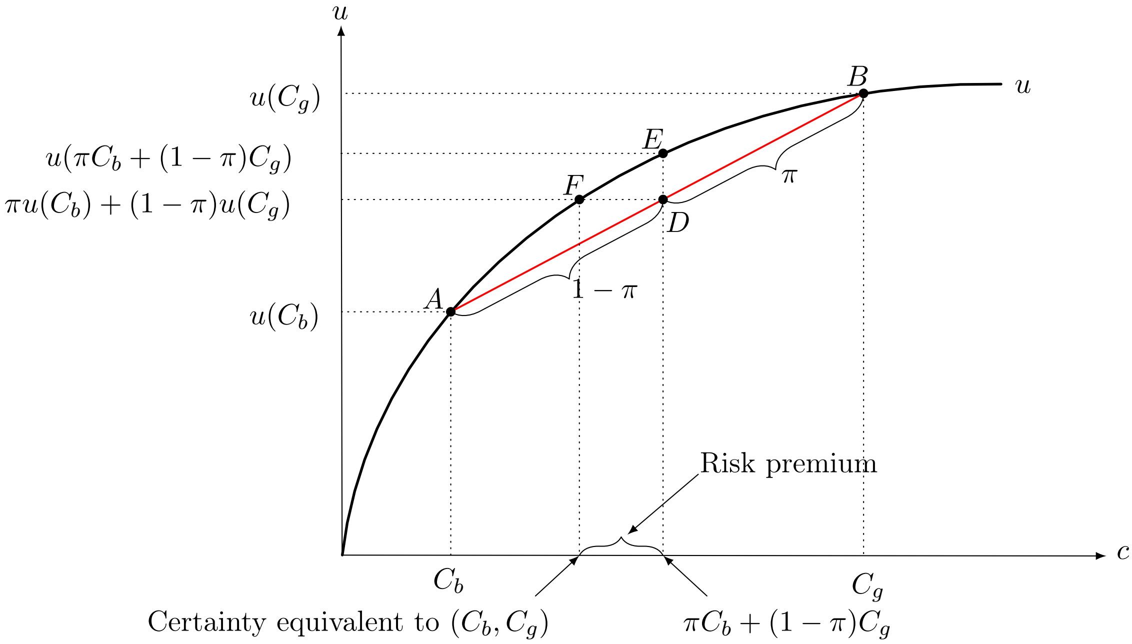
– Compare (c_b, c_g) and $(\pi c_b + (1-\pi)c_g, \pi c_b + (1-\pi)c_g)$

Example. $c_b = 5, c_g = 15$ and $\pi = 0.5$

– Which one does the consumer prefers?

$$\begin{pmatrix} \text{Risk-loving} \\ \text{Risk-neutral} \\ \text{Risk-averse} \end{pmatrix} \Leftrightarrow \pi u(c_b) + (1-\pi)u(c_g) \begin{pmatrix} > \\ = \\ < \end{pmatrix} u(\pi c_b + (1-\pi)c_g \Leftrightarrow u : \begin{pmatrix} \text{Convex} \\ \text{Linear} \\ \text{Concave} \end{pmatrix}$$

Example (continued)**.** $u(c) = \sqrt{c}$: concave $\rightarrow 0.5\sqrt{5} + 0.5\sqrt{15} < \sqrt{10}$



Risk Averse Consumer

– Diversification and risk-spreading

Example. Suppose that someone has to carry 16 eggs using 2 baskets with each basket likely to be broken with half probability:

$$\begin{cases} \text{Carry all eggs in one basket} : 0.5u(16) + 0.5u(0) = 2 \\ \text{Carry 8 eggs in each of 2 baskets} : 0.25u(16) + 0.25u(0) + 0.5u(8) = 1 + \sqrt{2} \end{cases}$$

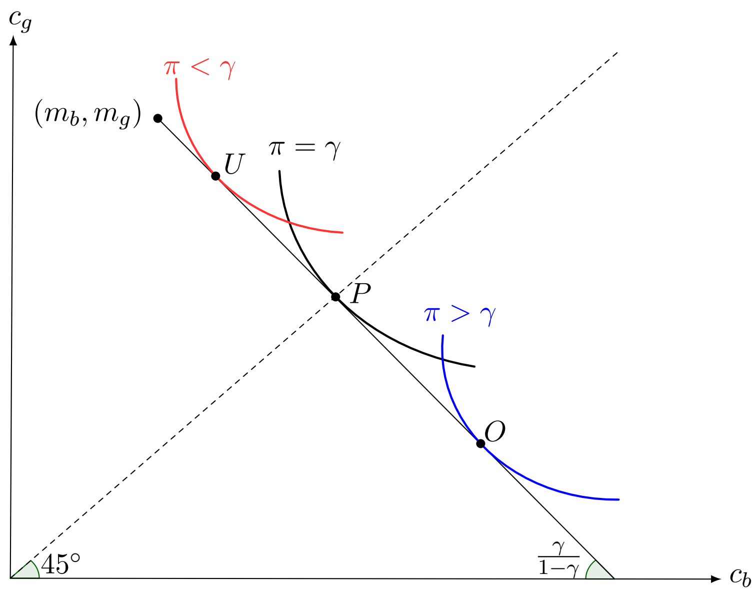
$\rightarrow$ "Do not put all your eggs in one basket"

III. Choice

– Maximize $\pi u(c_b) + (1-\pi)u(c_g)$ subject to $c_b + \frac{(1-\gamma)}{\gamma}c_g = m_b + \frac{(1-\gamma)}{\gamma}m_g$

– Tangency condition:

$$\frac{\pi u'(c_b)}{(1-\pi)u'(c_g)} = \frac{MU_b}{MU_g} = \text{slope of budget line} = \frac{\gamma}{1-\gamma}$$



– $\begin{pmatrix} \pi > \gamma \rightarrow c_b > c_g : & \text{Over-insured} \\ \pi < \gamma \rightarrow c_b < c_g : & \text{Under-insured} \\ \pi = \gamma \rightarrow c_b = c_g : & \text{Perfectly-insured} \end{pmatrix}$

– The rate $\gamma = \pi$ is called 'fair' since the insurance company breaks even at that rate, or what it earns, γK, is equal to what it pays, $\pi K + (1-\pi)0 = \gamma K$.

Ch. 14. Consumer Surplus

I. Measuring the change in consumer welfare

A. Let $\Delta CS \equiv$ change in the consumer surplus due to the price change $p_1 \to p_1' > p_1$

– This is a popular method to measure the change in consumer welfare

– The idea underlying this method is that the demand curves measures the consumer's willingness to pay.

B. This works perfectly in case of the quasi linear utility: $u(x_1, x_2) = v(x_1) + x_2$

– Letting $p_2 = 1$ and assuming a tangent solution,

$$v'(x_1) = MRS = p_1 \Rightarrow x_1(p_1): \text{ demand function}$$

– So the demand curve gives a correct measure of the consumer's WTP and thus ΔCS measures the change in the consumer welfare due to the price change.

– To verify, let $x_1 \equiv x_1(p_1)$ and $x_1' = x_1(p_1')$,

$$\begin{aligned}\Delta CS &= (p_1' - p_1)x_1' + \textstyle\int_{x_1'}^{x_1}[v'(s) - p_1]ds \\ &= (p_1' - p_1)x_1' + v(x_1) - v(x_1') - p_1(x_1 - x_1') \\ &= [v(x_1) + m - p_1x_1] - [v(x_1') + m - p_1'x']\end{aligned}$$

– However, this only works with the quasi linear utility.

C. Compensating and equivalent variation (CV and EV)

– Idea: How much income would be needed to achieve a given level utility for consumer under difference prices?

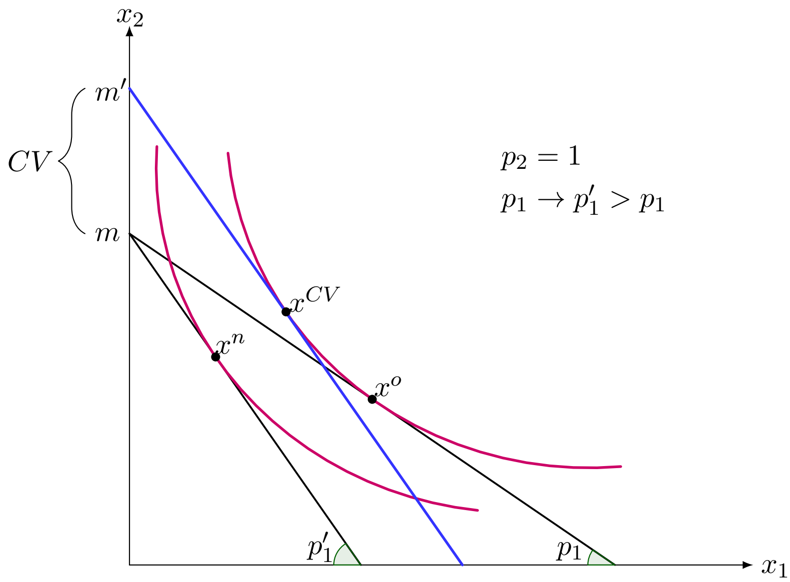
– Let $\begin{cases} p^o \equiv (p_1, 1), \; x^o \equiv x(p^o, m), \text{ and } u^o \equiv u(x^o) \\ p^n \equiv (p_1', 1), \; x^n \equiv x(p^n, m), \text{ and } u^n \equiv u(x^n) \end{cases}$

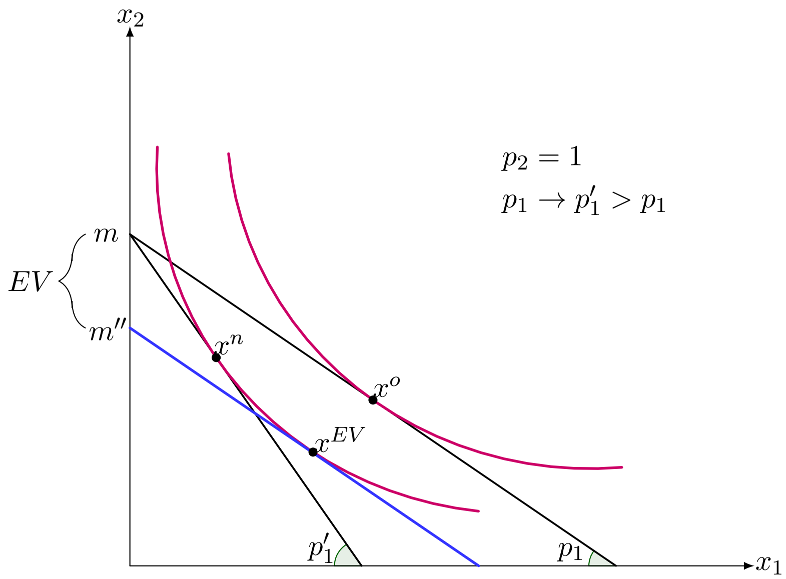
– Calculate m' = income needed to attain u^o under p^n, that is $u^o = u(x(p^o, m'))$

$\rightarrow$ define CV $\equiv |m - m'|$ and say that consumer becomes worse(better) off as much as CV if $m < (>) m'$.

– Calculate m'' = income needed to attain u^n under p^o, that is $u^n = u(x(p^o, m''))$

$\rightarrow$ define EV $\equiv |m - m''|$ and say that consumer becomes worse(better) off as much as EV if $m > (<) m''$.





Example. $\begin{cases} p^o = (1,1) \\ p^n = (2,1) \end{cases}$, $m = 100, \; u(x_1, x_2) = x_1^{1/2} x_2^{1/2} \rightarrow \begin{cases} u^0 = u(50, 50) = 50 \\ u^n = u(50, 25) = 25\sqrt{2} \end{cases}$

(i) $p^n = (2,1) \,\&\, m' \rightarrow (\frac{m'}{4}, \frac{m'}{2}), \; u(\frac{m'}{4}, \frac{m'}{2}) = \frac{m'}{2\sqrt{2}} = 50, \;$ so $m' \approx 141$

(ii) $p^o = (1,1) \,\&\, m'' \rightarrow (\frac{m''}{2}, \frac{m''}{2}), u(\frac{m''}{2}, \frac{m''}{2}) = \frac{m''}{2} = 25\sqrt{2}, \;$ so $m'' \approx 70$

– In general, ΔCS is in between CV and EV

– With quasi-linear utility, we have $\Delta CS = CV = EV$

$$u^o = v(x_1) + m - p_1x_1 = v(x_1') + m' - p_1'x_1'$$
$$\to CV = |m' - m|$$
$$= |(v(x_1) - p_1x_1) - (v(x_1') - p_1'x_1')|$$

$$u^n = v(x_1') + m - p_1'x_1' = v(x_1) + m'' - p_1x_1$$
$$\to EV = |m - m''|$$
$$= |(v(x_1) - p_1x_1) - (v(x_1') - p_1'x_1')|$$

II. Producer's surplus and benefit-cost analysis

A. Producer's surplus: Area between price and supply curve

Ch. 15. Market Demand

I. Individual and market demand

– n consumers in the market

– Consumer i's demand for good k: $x_k^i(p, m_i)$

– Market demand for good k: $D_k(p, m_1, \cdots, m_n) = \sum_{i=1}^n x_k^i(p, m_i)$

Example. Suppose that there are 2 goods and 2 consumers who have demand for good 1 as follows: With $p_2, m_1,$ and, m_2 being fixed,

$$x_1^1(p_1) = \max\{20 - p_1, 0\} \text{ and } x_1^2(p_1) = \max\{10 - 2p_1, 0\}$$

II. Demand elasticity

A. Elasticity (ε): Measures a responsiveness of one variable y to another variable x

$$\varepsilon = \frac{\Delta y/y}{\Delta x/x} = \frac{\%\text{ change of } y \text{ variable}}{\%\text{ change of } x \text{ variable}}$$

B. Demand elasticity: $\varepsilon_p = -\frac{\Delta D(p)/D(p)}{\Delta p/p} = -\frac{\Delta D(p)}{\Delta p}\frac{p}{D(p)}$, where $\Delta D(p) = D(p + \Delta p) - D(p)$

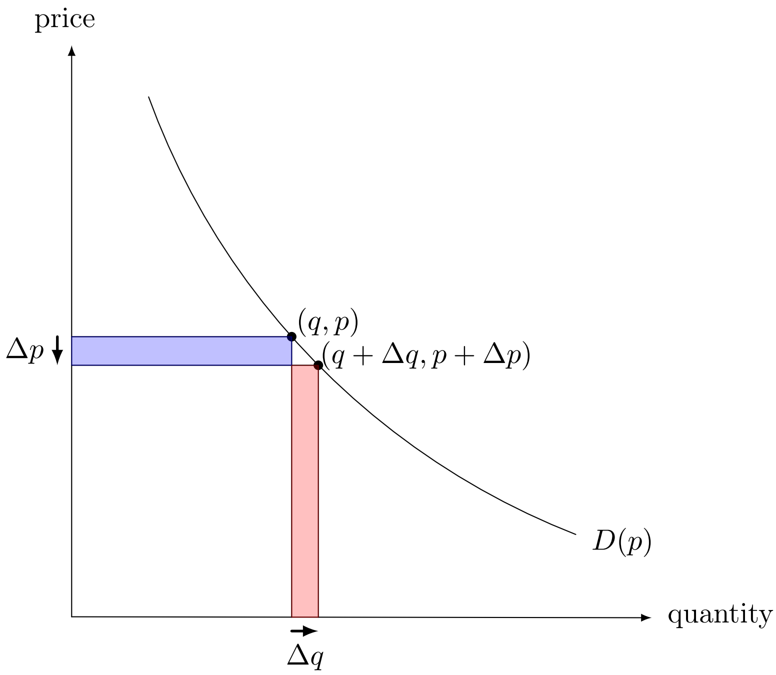
– As $\Delta p \to 0$, we have $\varepsilon_p = -\frac{dD(p)}{dp}\frac{p}{D(p)} = -\frac{pD'(p)}{D(p)}$, (point elasticity)

Example. Let $D(p) = Ap^{-b}$, where A, $b > 0 \to \varepsilon_p = b$ or constant

C. Demand elasticity and marginal revenue:

– Revenue: $R = D(p)p = D^{-1}(q)q$, where $q = D(p)$

– Marginal revenue: Rate of change in revenue from selling an extra unit of output



$$MR = \frac{\Delta R}{\Delta q} = \frac{{\color{blue}q\Delta p} + {\color{red}(p+\Delta p)\Delta q}}{\Delta q} \simeq \frac{q\Delta p + p\Delta q}{\Delta q} = p\left[1 + \frac{D(p)}{p}\frac{\Delta p}{\Delta D(p)}\right] = p\left[1 - \frac{1}{\varepsilon_p}\right].$$

– Revenue increases (decreases) or $MR > 0 (< 0)$ if $\varepsilon_p > 1 (< 1)$

Example. Let $D(p) = A - bp$, where A, $b > 0 \rightarrow \varepsilon_p = \frac{bp}{A-bp}$.

Ch. 18. Technology

I. Production technology

– Inputs: labor, land, capital (financial or physical), and raw material

– Output(s)

– Production set: All combinations of inputs and outputs that are technologically feasible

A. Production function: A function describing the boundary of production set

– Mathematically,

$$y = f(x),$$

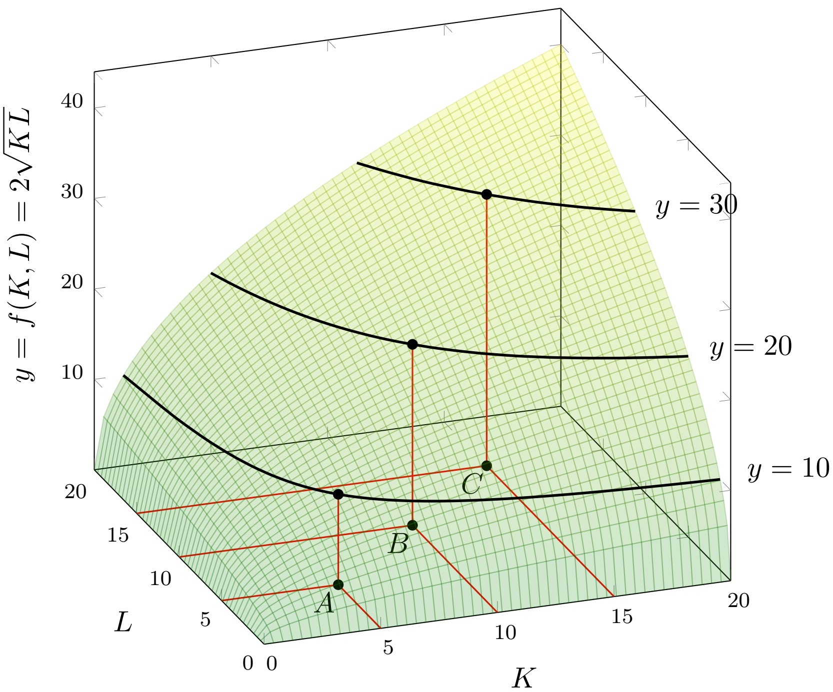
where x =amount of input(s), y =amount of ouput

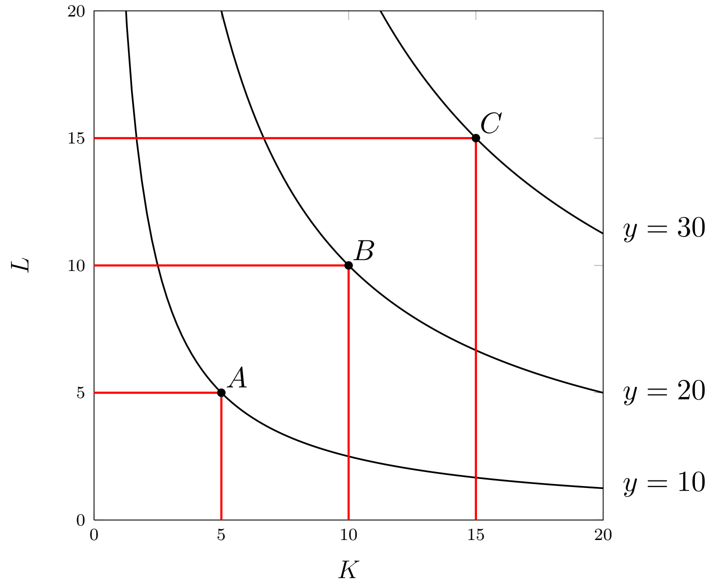
– Two prod. functions do not represent the same technology even if one is a monotone transformation of the other.

– From now on, we (mostly) assume that there are 2 inputs and 1 output.

B. Isoquant: Set of all possible combinations of input 1 and 2 that yields the same level of output, that is

$$Q(y) \equiv \{(x_1, x_2) | f(x_1, x_2) = y\} \text{ for a given } y \in \mathbb{R}_+$$





C. Marginal product and technical rate of transformation

– Marginal product of input 1: $MP_1 = \frac{\Delta y}{\Delta x_1} = \frac{f(x_1+\Delta x_1,x_2)-f(x_1,x_2)}{\Delta x_1}$

– Technical rate of transformation: Slope of isoquant

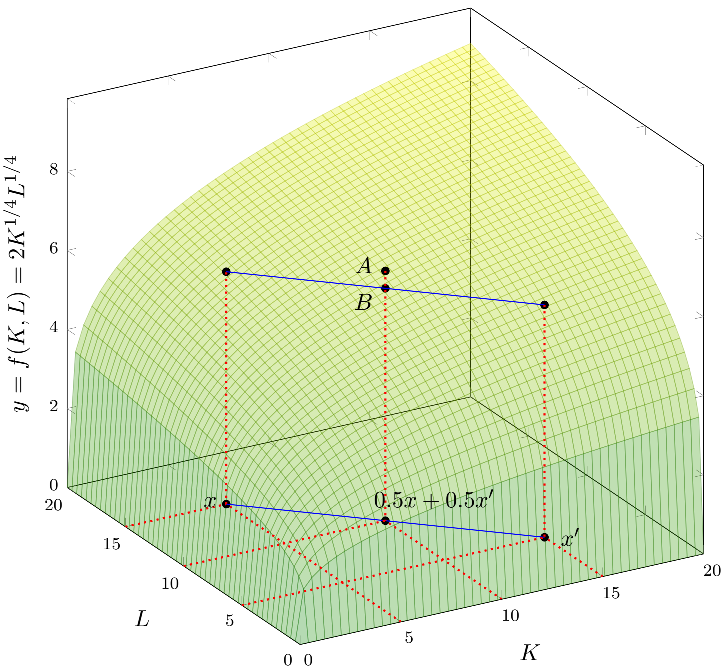
$$TRS = \left|\frac{\Delta x_2}{\Delta x_1}\right| = \frac{MP_1}{MP_2} \leftarrow \Delta y = MP_1 \Delta x_1 + MP_2 \Delta x_2 = 0$$

D. Examples: Perfect complements, perfect substitutes, Cobb-Douglas

II. Desirable properties of technology

– Monotonic: f is increasing in x_1 and x_2

– Concave: $f(\lambda x + (1-\lambda)x') \geq \lambda f(x) + (1-\lambda) f(x')$ for $\lambda \in [0,1]$



– Decreasing MP: Each MP_i is decreasing with x_i

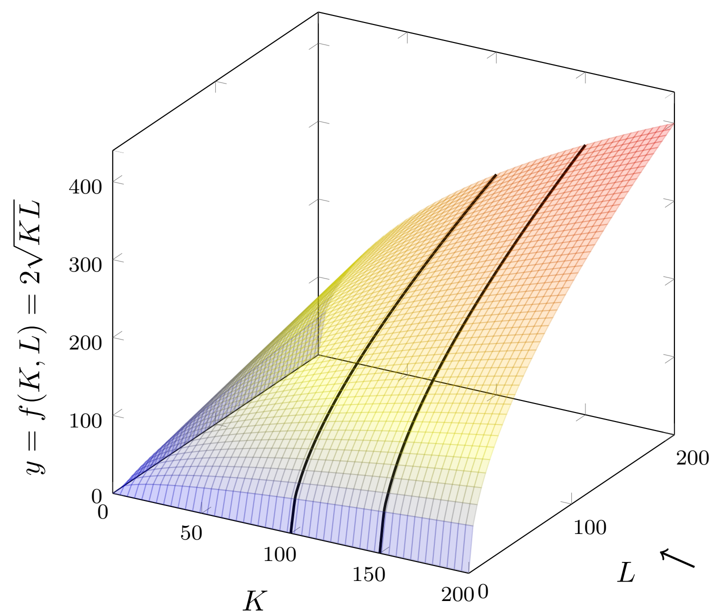
– Decreasing TRS: TRS is decreasing with x_1

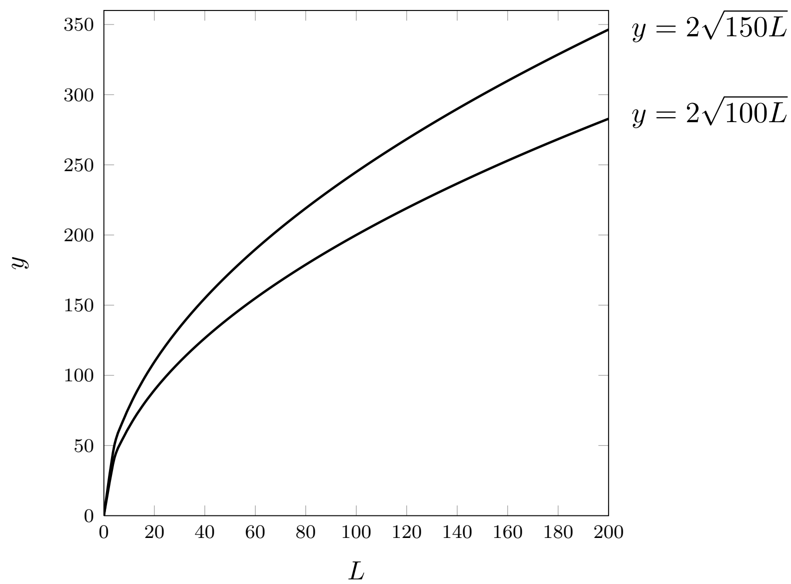
III. Other concepts

A. Long run and short run

– Short run: Some factors are fixed $\leftrightarrow$ Long run: All factors can be varying

Example. What is the production function if factor 2, say capital, is fixed in the SR?





B. Returns to scale: How much output increases as all inputs are scaled up simultaneously?

$$f(tx_1, tx_2) \begin{pmatrix} > \\ = \\ < \end{pmatrix} tf(x_1, x_2) \leftrightarrow \begin{pmatrix} \text{increasing} \\ \text{constant} \\ \text{decreasing} \end{pmatrix} \text{ returns to scale}$$

Example. $$\left(\begin{array}{l} \text{IRS} : f(x) = x_1 x_2 \\ \text{CRS} : f(x) = \min\{x_1, x_2\} \\ \text{DRS} : f(x) = \sqrt{x_1 + x_2} \end{array} \right.$$

What about $f(x) = K x_1^a x_2^b$?

Ch. 19. Profit Maximization

I. Profits

– p = price of output; w_i = price of input i

– Profit is total revenue minus total cost:

$$\pi = py - \sum_{i=1}^{2} w_i x_i$$

– Non-profit goals? ← Separation of ownership and control in a corporation

II. Short-run profit maximization

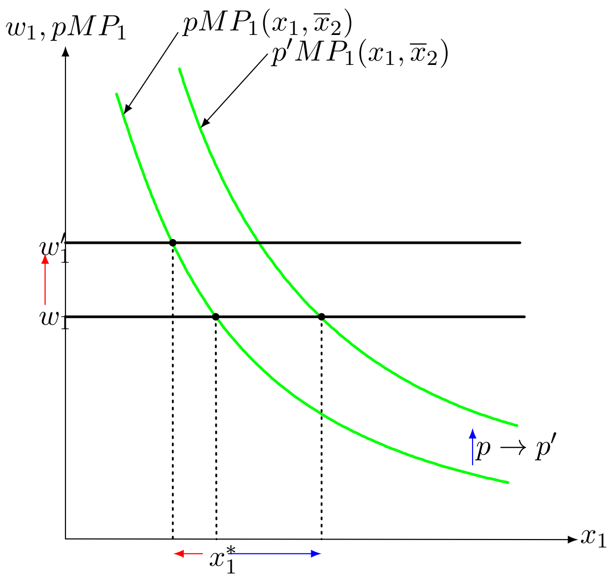
– With input 2 fixed at $\bar{x}_2$,

$$\max_{x_1} \; pf(x_1, \bar{x}_2) - w_1 x_1 - w_2 \bar{x}_2$$

$$\xrightarrow{F.O.C.} \; pMP_1(x_1^*, \bar{x}_2) = w_1$$

that is, the value of marginal product of input 1 equals its price

– Graphically,



– Comparative statics: Demand of a factor $\begin{pmatrix} \text{decreases} \\ \text{increases} \end{pmatrix}$ with $\begin{pmatrix} \text{its own price} \\ \text{output price} \end{pmatrix}$

– Factor demand: the relationship between the demand of a factor and its price

III. Long-run profit maximization

– All inputs are variable

$$\max_{x_1, x_2} pf(x_1, x_2) - w_1 x_1 - w_2 x_2$$

$$\xrightarrow{F.O.C.} \begin{cases} pMP_1(x_1^*, x_2^*) = w_1 \\ pMP_2(x_1^*, x_2^*) = w_2 \end{cases}$$

– Comparative statics: (y, x_1, x_2) chosen under $(p, w_1, w_2) \to (y', x_1', x_2')$ chosen under (p', w_1', w_2')

1) Profit maximization requires

$$\begin{array}{rl} & py - w_1 x_1 - w_2 x_2 \geq py' - w_1 x_1' - w_2 x_2' \\ + & p'y' - w_1' x_1' - w_2' x_2' \geq p'y - w_1' x_1 - w_2' x_2 \\ \hline & (p' - p)(y' - y) - (w_1' - w_1)(x_1' - x_2) - (w_2' - w_2)(x_2' - x_2) \geq 0 \end{array}$$

$$\to \Delta p \Delta y - \Delta w_1 \Delta x_1 - \Delta w_2 \Delta x_2 \geq 0$$

2) $\Delta w_1 = \Delta w_2 = 0 \to \Delta p \Delta y \geq 0$, or the supply of output increases with its price

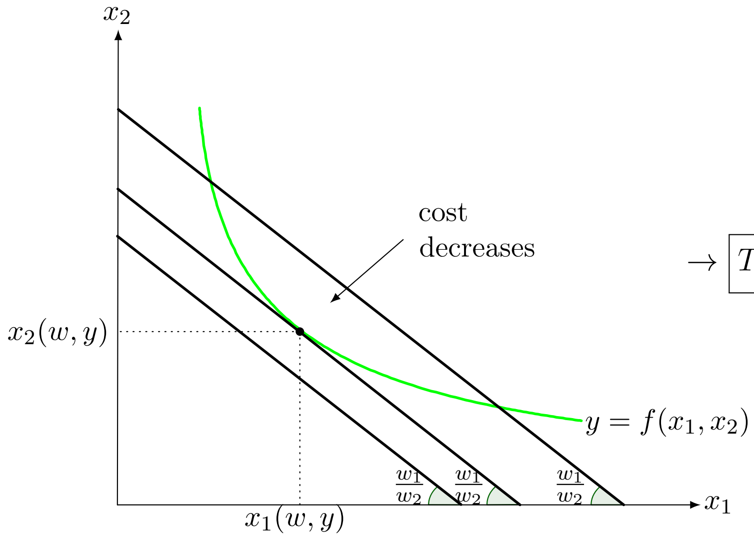
3) $\Delta p = \Delta w_2 = 0 \to \Delta w_1 \Delta x_1 \leq 0$, or the demand of input decreases with its price

Ch. 20. Cost Minimization

I. Cost minimization: Minimize the cost of producing a given level of output y

$$\min_{x_1,x_2} w_1x_1 + w_2x_2 \text{ subject to } (x_1, x_2) \in Q(y) \text{ (i.e. } f(x_1, x_2) = y)$$

A. Tangent solution: Consider *iso-cost line* for each cost level C, $w_1x_1 + w_2x_2 = C$; and find the *lowest* iso-cost line that meets the isoquant curve



$\rightarrow \boxed{TRS = \frac{MP_1}{MP_2} = \frac{w_1}{w_2}}$

$$\rightarrow \begin{cases} x_1(w,y), x_2(w,y) : \text{ Conditional factor demand function} \\ c(w,y) = w_1x_1(w,y) + w_2x_2(w,y) : \text{ Cost function} \end{cases}$$

B. Examples

– Perfect complement: $y = \min\{x_1, x_2\}$

$$\rightarrow x_1(w,y) = x_2(w,y) = y$$
$$c(w,y) = w_1x_1(w,y) + w_2x_2(w,y) = (w_1 + w_2)y$$

– Perfect substitutes: $y = x_1 + x_2$

$$\rightarrow x(w,y) = \begin{cases} (y,0) & \text{if } w_1 < w_2 \\ (0,y) & \text{if } w_2 < w_1 \end{cases}$$
$$c(w,y) = \min\{w_1, w_2\}y$$

– Cobb-Douglas: $y = Ax_1^a x_2^b \rightarrow \begin{cases} TRS = \frac{ax_2}{bx_1} = \frac{w_1}{w_2} \\ y = Ax_1^a x_2^b \end{cases}$

$$\rightarrow c(w,y) = Kw_1^{\frac{a}{a+b}} w_2^{\frac{b}{a+b}} y^{\frac{1}{a+b}}, \text{ where } K \text{ is a constant depending on } a,\ b, \text{ and } A$$

II. Comparative statics: (x_1, x_2) under $(w_1, w_2, y) \to (x_1', x_2')$ under (w_1', w_2', y)

– Cost minimization requires: Letting $\Delta x_1 \equiv x_1' - x_1$ and so on,

$$\begin{array}{rl} & w_1 x_1' + w_2 x_2' \geq w_1 x_1 + w_2 x_2 \\ + & \underline{\left| \, w_1' x_1 + w_2' x_2 \geq w_1' x_1' + w_2' x_2' \qquad\qquad\qquad\quad \right.} \\ & -(w_1' - w_1)(x_1' - x_2) - (w_2' - w_2)(x_2' - x_2) \geq 0 \quad \to \quad \Delta w_1 \Delta x_1 + \Delta w_2 \Delta x_2 \leq 0 \end{array}$$

– Setting $\Delta w_2 = 0$, we obtain $\Delta w_1 \Delta x_1 \leq 0$, or the conditional demand of an input falls as its price rises

III. Average cost and returns to scale

– Average cost: $AC(y) = \dfrac{c(w_1, w_2, y)}{y}$, that is per-unit-cost to produce y units of output

– Assuming DRS technology, consider two output levels y^0 and $y^1 = ty^0$ with $t > 1$: For any $x \in Q(y^1)$,

$$tf\left(\frac{x}{t}\right) > f\left(t\frac{x}{t}\right) \text{ or } f\left(\frac{x}{t}\right) > \frac{f(x)}{t} = \frac{y^1}{t} = y^0,$$

which implies

$$c(w, y^0) < w_1 \frac{x_1}{t} + w_2 \frac{x_2}{t} = \frac{1}{t}(\, w_1 x_1 + w_2 x_2 \,) \text{ for all } x \in Q(y^1).$$

So

$$c(w, y^0) < \frac{1}{t} \, c(w, y^1) \, = \frac{y^0}{y^1} c(w, y^1) \text{ or}$$

$$AC(y^0) = \frac{c(w, y^0)}{y^0} < \frac{c(w, y^1)}{y^1} = AC(y^1)$$

– Applying a similar argument to IRS or CRS technology, we obtain

$$\begin{cases} \text{DRS} : AC(y) \text{ is increasing} \\ \text{IRS} : AC(y) \text{ is decreasing} \\ \text{CRS} : AC(y) \text{ is constant} \ \to \ AC(y) = c(w_1, w_2) \text{ or } c(w, y) = c(w_1, w_2)y \end{cases}$$

IV. Long run and short run cost

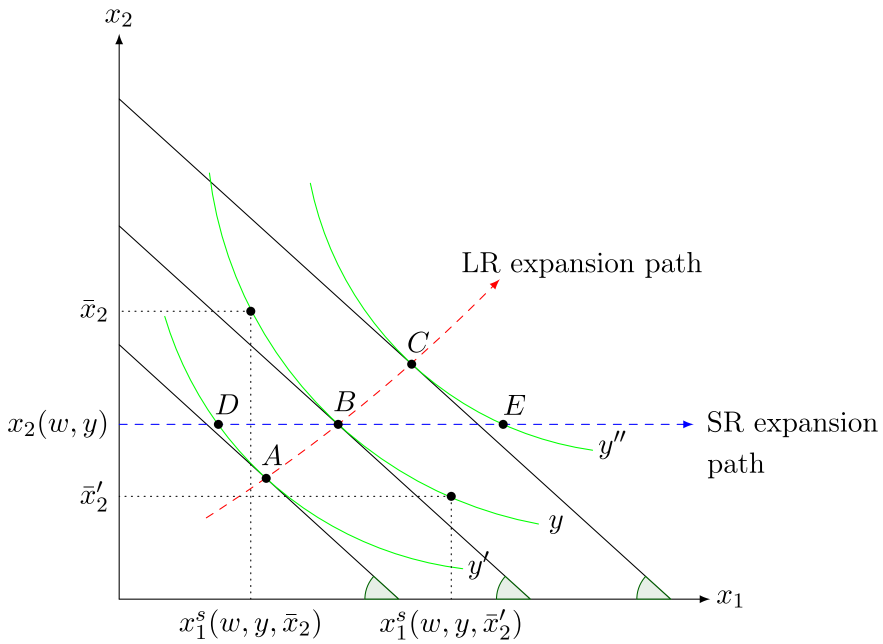
– Suppose that input 2 is fixed at $\bar{x}_2$ in the SR

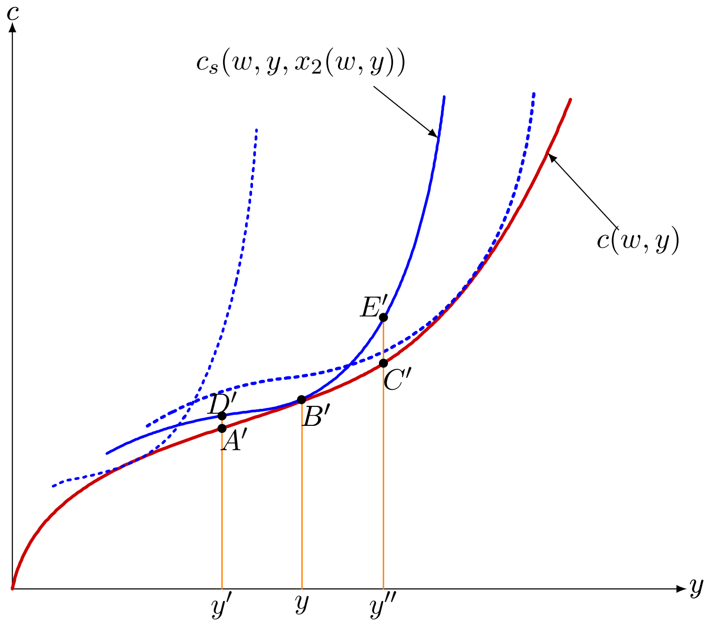
A. Short run demand and cost

– SR demand: $x_1^s(w, y, \bar{x}_2)$ satisfying $f(x_1^s(w, y, \bar{x}_2), \bar{x}_2) = y$

– SR cost: $c_s(w, y, \bar{x}_2) = w_1 x_1^s(w, y, \bar{x}_2) + w_2 \bar{x}_2$

– Envelope property: $c(w,y) = c_s(w,y,x_2(w,y)) = \min_{x_2} c_s(w,y,x_2) \leq c_s(w,y,\bar{x}_2)$





Example. $f(x) = x_1 x_2$

$\rightarrow x_1^s = \frac{y}{\bar{x}_2}$ and $c_s = w_1 x_1^s + w_2\bar{x}_2 = w_1\frac{y}{\bar{x}_2} + w_2\bar{x}_2$

$\rightarrow c(w,y) = \min_{\bar{x}_2} \; w_1\frac{y}{\bar{x}_2} + w_2\bar{x}_2$

Ch. 21. Cost Curves

I. Various concepts of cost

– Suppose that $c(y) = c_v(y) + F,\ c_v(0) = 0$

A. Average costs:

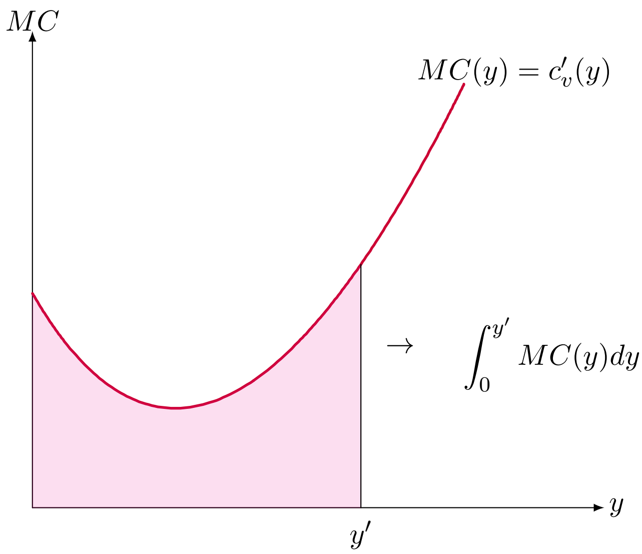
$$AC(y) = \frac{c(y)}{y} = \frac{c_v(y) + F}{y} = \frac{c_v(y)}{y} + \frac{F}{y} = AVC(y) + AFC(y)$$

B. Marginal cost:

$$MC(y) = \lim_{\Delta y \to 0} \frac{c(y + \Delta y) - c(y)}{\Delta y} = c'(y) = c_v'(y)$$

II. Facts about cost curves

– The area below MC curve = variable cost



$$\rightarrow \int_0^{y'} MC(y)dy = \int_0^{y'} c_v'(y)dy$$
$$= c_v(y')$$

– MC and AVC curves start at the same point

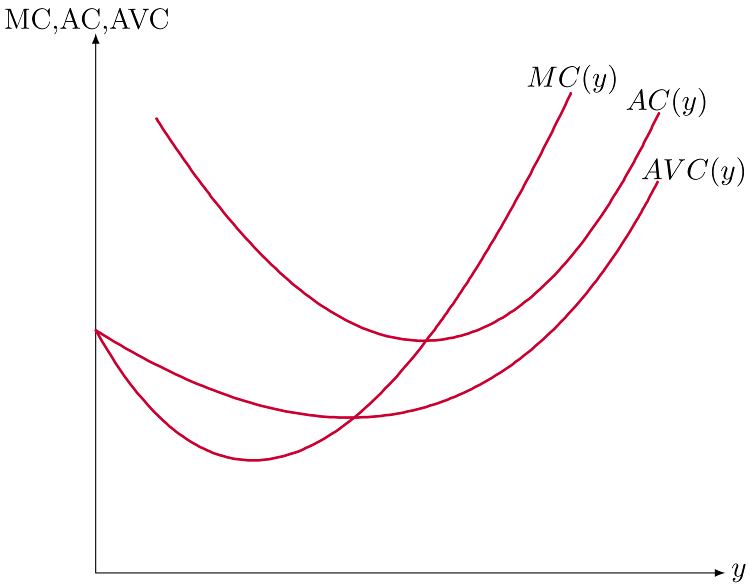
$$MC(0) = \lim_{\Delta y \to 0} \frac{c_v(\Delta y) - c_v(0)}{\Delta y} = \lim_{\Delta y \to 0} \frac{c_v(\Delta y)}{\Delta y} = AVC(0)$$

– AC is decreasing (increasing) when MC is below (above) AC

$$\frac{d}{dy}AC(y) = \frac{d}{dy}\left(\frac{c(y)}{y}\right) = \frac{c'(y)y - c(y)}{y^2} = \frac{1}{y}\left(c'(y) - \frac{c(y)}{y}\right)$$
$$= \frac{1}{y}\left(MC(y) - AC(y)\right) > 0 \text{ if } MC(y) > AC(y)$$
$$< 0 \text{ if } MC(y) < AC(y)$$

$\rightarrow$ The same facts hold for AVC

– As a result, we have



– Example: $c(y) = \frac{1}{3}y^3 - y^2 + 2y + 1$

$$\rightarrow \begin{cases} MC(y) = y^2 - 2y + 2 \\ AVC(y) = \frac{1}{3}y^2 - y + 2 \\ AC(y) = \frac{1}{3}y^2 - y + 2 + \frac{1}{y} \end{cases}$$

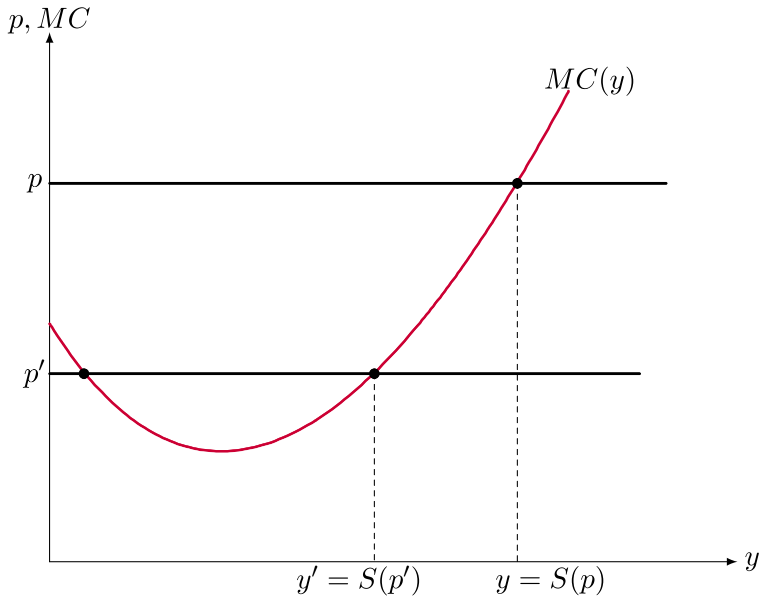
Ch. 22. Firm Supply

I. Supply decision of a competitive firm

A. Given cost function, $c(y)$, the firm maximizes its profit

$$\max_y py - c(y)$$

$\xrightarrow{F.O.C.} p = MC(y)$, which yields the supply function, $y = S(p)$.



B. Two caveats: Assume that $c(y) = c_v(y) + F$, where the firm cannot avoid incurring the fixed cost F in the short run while it can in the long run

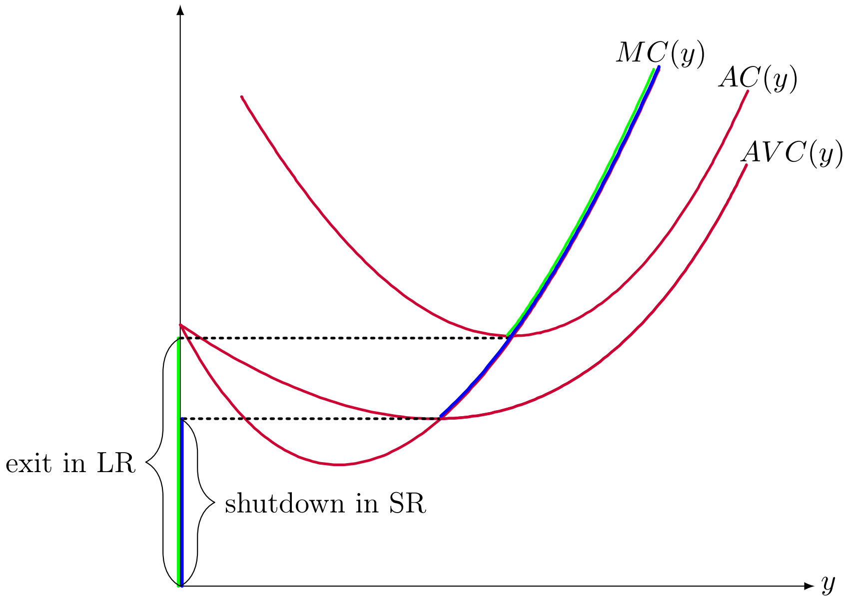
1. The solution must be at the upward-sloping part of MC curve (2nd order condition)
2. Boundary solution where $y = 0$ is optimal: "Shutdown" in SR or "Exit" in LR
   – Shutdown means that the fixed cost has to be incurred anyway
   (i) SR: Shutdown is optimal if $p < \min_{y>0} AVC(y)$

$$\because \; p < \frac{c_v(y)}{y} \text{ for all } y > 0 \;\rightarrow\; py - c_v(y) - F < -F \text{ for all } y > 0$$

   (ii) LR: Exit is optimal if $p < \min_{y>0} AC(y)$

$$\because \; p < \frac{c(y)}{y} \text{ for all } y > 0 \;\rightarrow\; py - c(y) < 0 \text{ for all } y > 0$$

3. Supply curves



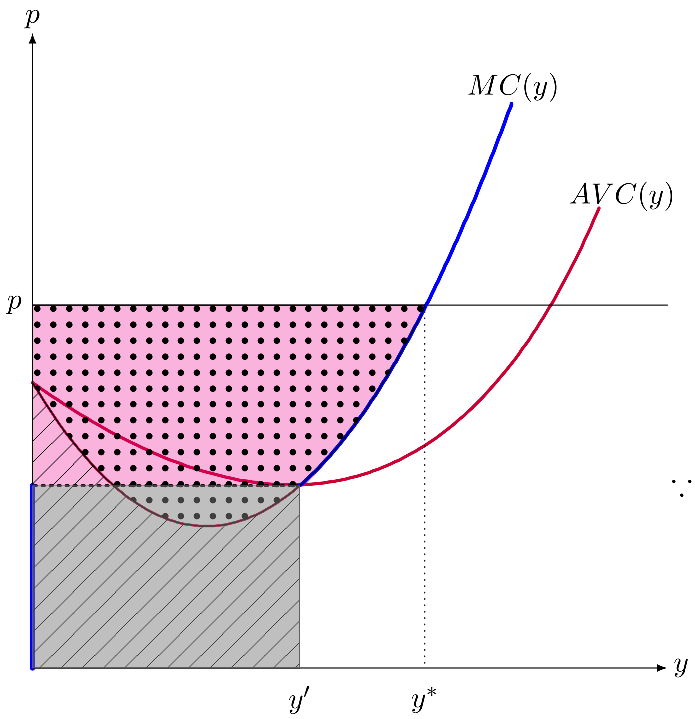
– PS: Accumulation of extra revenue minus extra cost from producing extra unit of output or

$$\text{PS} = \int_0^{y^*} (p - MC(y))dy = py^* - \int_0^{y^*} MC(y)dy = py^* - c_v(y^*)$$

– Thus,

$$\text{Profit} = py^* - c(y^*) = py^* - c_v(y^*) - F = \text{PS} - F$$

– Graphically,



Dotted area (=producer's surplus)

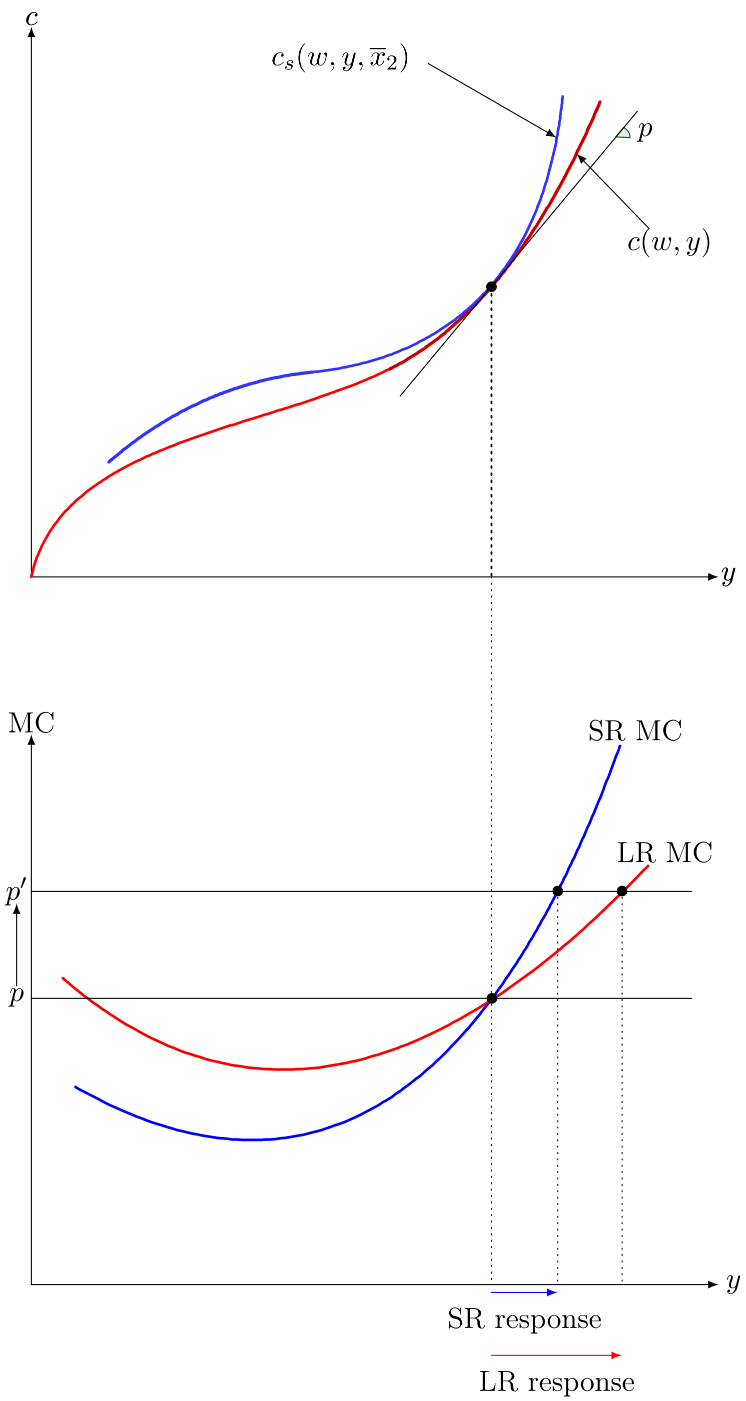
= Red area (=area b/w supply curve and price)

$\because$ Gray area $(= y' \times AVC(y'))$

= Stroked area $(= c_v(y'))$

C. Long-run and short-run firm supply

- Assume more generally that the SR cost is given as $c_s(w, y, \bar{x}_2)$ and the LR cost as $c(w, y)$.
- The envelope property implies that the marginal cost curve is steeper in the LR than in the SR → The firm supply responds more sensitively to the price change in the LR than in the SR.



Ch. 23. Industry Supply

I. Short run industry supply

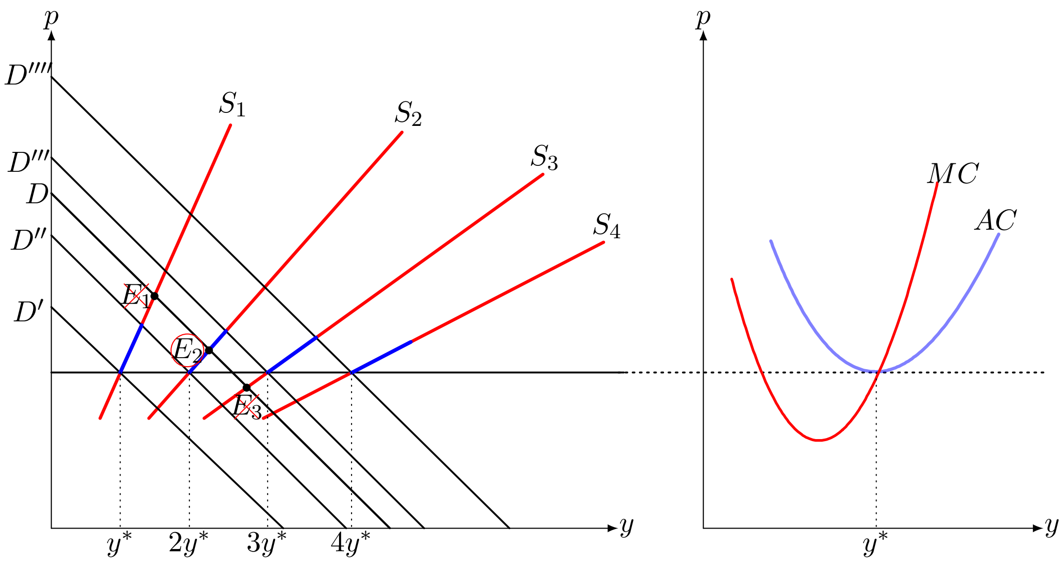
– Let $S_i(p)$ denote firm i's supply at price p. Then, the industry supply is given as

$$S(p) = \sum_{i=1}^{n} S_i(p).$$

→ Horizontal sum of individual firms' supply curves

II. Long run industry supply

– Due to free entry, $\begin{cases} \text{profit} \rightarrow \text{entry of firms} \rightarrow \text{lower price} \rightarrow \text{profit disappears} \\ \text{loss} \rightarrow \text{exit of firms} \rightarrow \text{higher price} \rightarrow \text{loss disappears} \end{cases}$



→ LR supply curve is (almost) horizontal

– LR equilibrium quantity y^* must satisfy $MC(y^*) = p = AC(y^*)$

– # of firms in the LR depends on the demand side

III. Fixed factors and economic rent

– No free entry due to limited resources (e.g. oil) or legal restrictions (e.g. taxicab license)

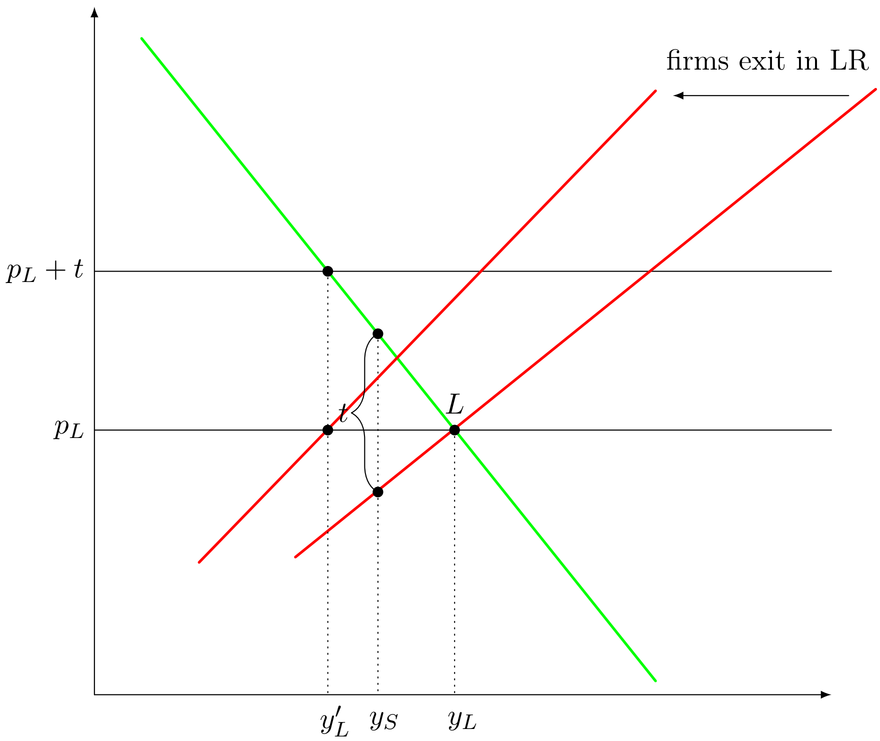
– Consider a coffee shop at downtown, earning positive profits even in the LR.

→ $py^* - c(y^*) \equiv F > 0$, which is the amount people would pay to rent the shop.

→ Economic profit is 0 since F is the economic rent (or opportunity cost)

IV. Application

A. Effects of quantity tax in SR and LR



B. Two-tier oil pricing

– Oil crisis in 70's: Price control on domestic oil

$$\rightarrow \begin{cases} \text{domestic oil at \$5/barell} : MC_1(y) \\ \text{imported oil at \$15/barell} : MC_2(y) = MC_1(y) + 10 \end{cases}$$

– $\bar{y}$ = the maximum amount of gasoline that can be produced using the domestic oil

– Due to the limited amount of domestic oil, the LR supply curve shifts up from $MC_1(y)$ to $MC_2(y)$ as y exceeds $\bar{y}$

Ch. 24. Monopoly

– A single firm in the market

– Set the price (or quantity) to maximize its profit

I. Profit maximization

A. Given demand and cost function, $p(y)$ and $c(y)$, the monopolist solves

$$\max_y \; p(y)y - c(y) = r(y) - c(y), \text{ where } r(y) \equiv p(y)y$$

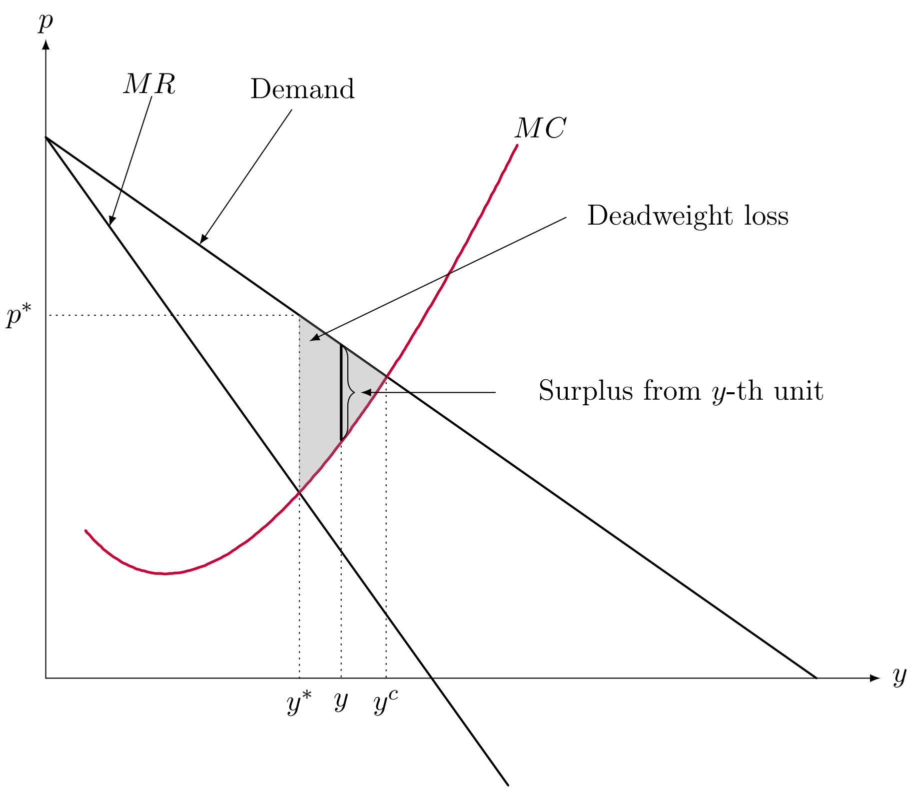
$$\begin{aligned} &\xrightarrow{F.O.C.} MR(y^*) = r'(y^*) = c'(y^*) = MC(y^*) \\ &\Leftrightarrow p(y^*) + yp'(y^*) = c'(y^*) \\ &\Leftrightarrow p(y^*)\left[1 + \tfrac{dp}{dy}\tfrac{y^*}{p(y^*)}\right] = c'(y^*) \\ &\Leftrightarrow p(y^*)\left[1 - \tfrac{1}{\varepsilon(y^*)}\right] = c'(y^*) > 0 \end{aligned}$$

So, $\varepsilon(y^*) > 1$, that is monopoly operates where the demand is elastic.

B. Mark-up pricing:

$$p(y^*) = \frac{MC(y^*)}{1 - 1/\varepsilon(y^*)} > MC(y^*), \text{ where } 1 - 1/\varepsilon(y^*): \text{ mark-up rate}$$

C. Linear demand example: $p = a - by \to MR(y) = \frac{d}{dy}\left(ay - 2by^2\right) = a - 2by$

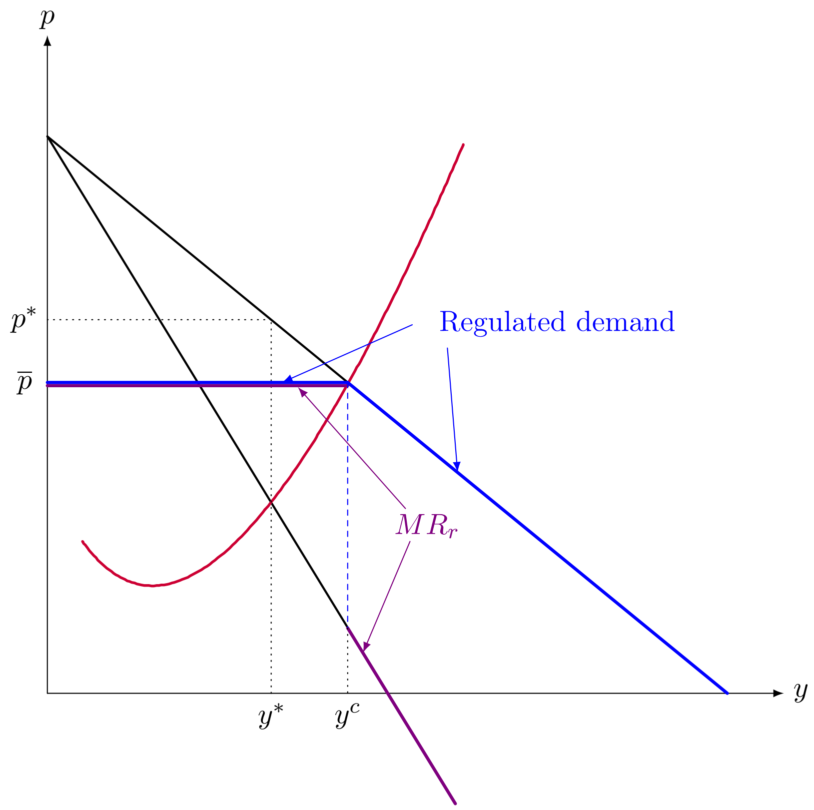


II. Inefficiency of monopoly

A. *Deadweight loss* problem: Decrease in quantity from y^c (the equilibrium output in the competitive market) to y^* reduces the social surplus

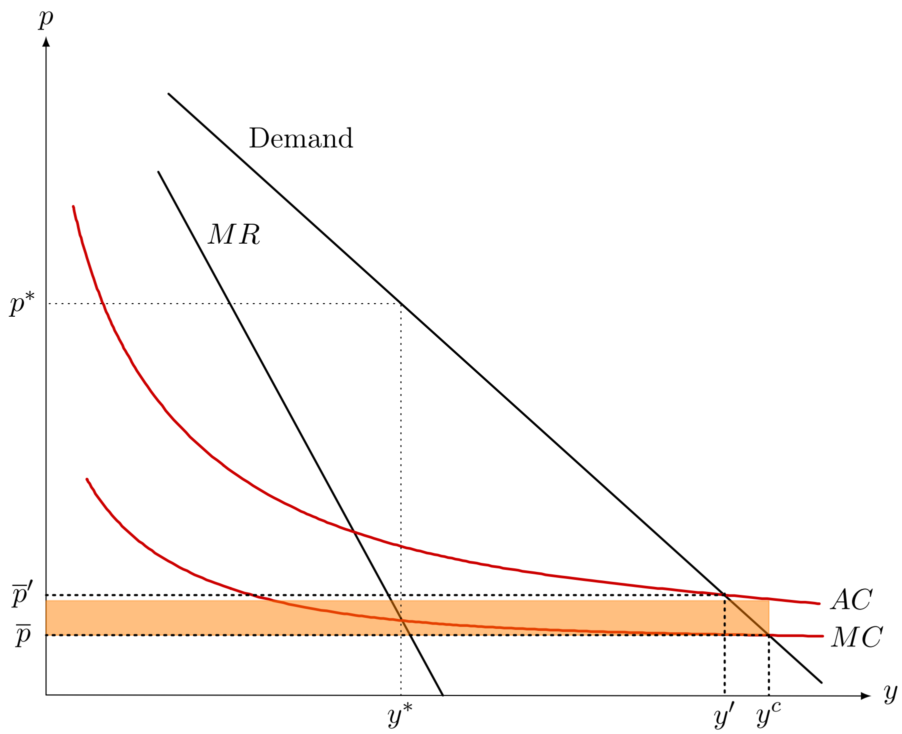
B. How to fix the deadweight loss problem

1. Marginal cost pricing: Set a price ceiling at $\bar{p}$ where MC = demand



$\rightarrow$ This will cause a natural monopoly to incur a loss and exit from the market

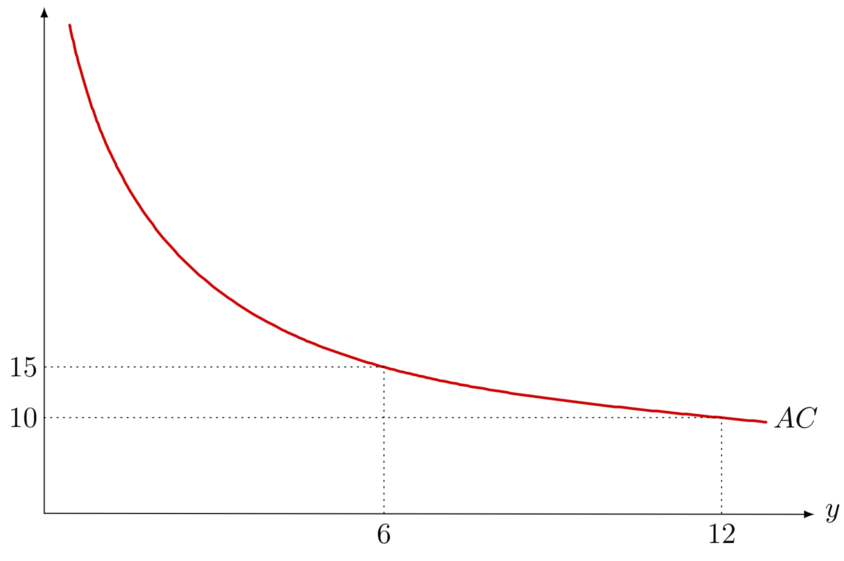
2. Average cost pricing: : Set a price ceiling at $\bar{p}'$ where AC = demand



$\rightarrow$ Not as efficient as MC pricing but no exit problem.

III. The sources of monopoly power

– Natural monopoly: Large minimum efficient scale relative to the market size



– An exclusive access to a key resource or right to sell: For example, DeBeer diamond, patents, copyright

– Cartel or entry-deterring behavior: Illegal

Ch. 25. Monopoly Behavior

– So far, we have assumed that the monopolist charges all consumers the same uniform price for each unit they purchase.

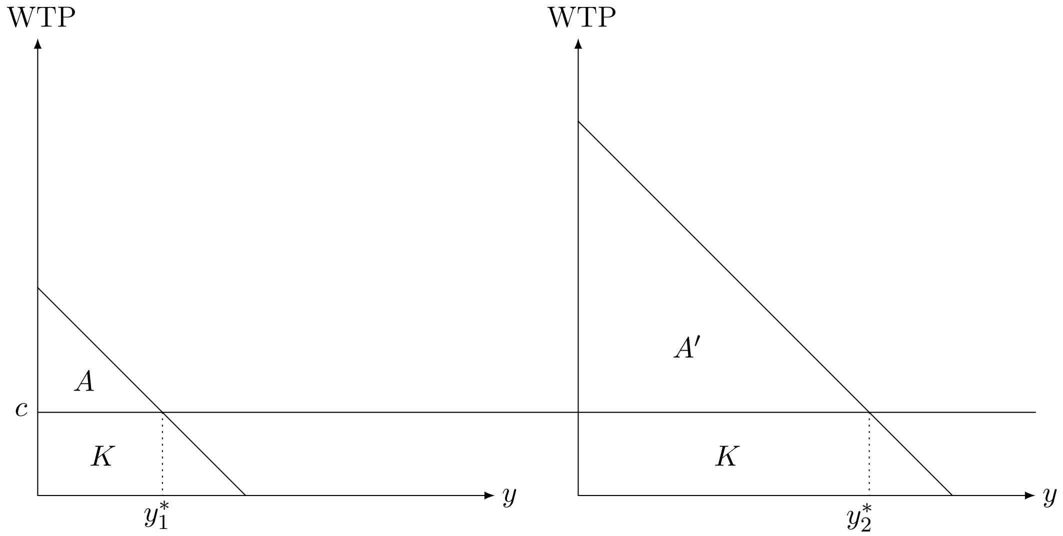
– However, it could charge $\begin{cases} \text{Different prices to different consumers (e.g. movie tickets)} \\ \text{Different per-unit-prices for different units sold (e.g. bulk discount)} \end{cases}$

– Assume $MC = c$(constant) and no fixed cost.

I. First-degree price discrimination (1° PD)

– The firm can charge different prices to different consumers and for different units.

– Two types of consumer with the following WTP (or demand)



– Take-it-or-leave offer: Bundle $(y_1^*, A + K)$ to type 1; bundle $(y_2^*, A' + K')$ to type 2

$\rightarrow$ No quantity distortion and no deadweight loss

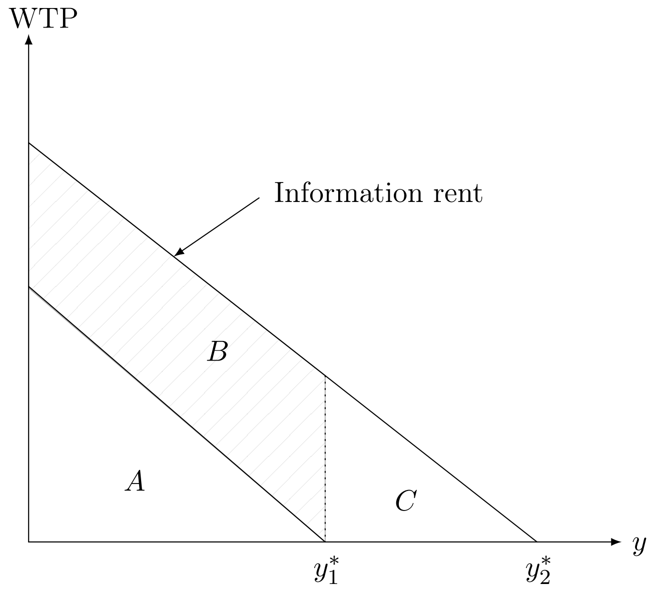
$\rightarrow$ Monopolist gets everything while consumers get nothing

– The same outcome can be achieved via "Two-Part Tariff": For type 1, for instance, charge A as an "entry fee" and c as "price per unit".

II. Second-degree discrimination: Assume $c = 0$ for simplicity

– Assume that the firm cannot tell who is what type

– Offer a *menu* of two bundles from which each type can *self select*



A. A menu which contains two bundles in the 1° PD does *not* work:

$$\text{Offer} \begin{cases} (y_1^*, A) \\ (y_2^*, A+B+C) \end{cases} \rightarrow \quad \begin{array}{cc} \text{What 1 gets} & \text{What 2 gets} \\ 0 & B \\ -B-C & 0 \end{array}$$

$\rightarrow$ Both types, in particular type 2, would like to select (y_1^*, A).

B. How to obtain the optimal menu:

1. From A, one can see that in order to sell y_2^* to type 2, the firm needs to reduce 2's payment:

$$\text{Offer} \begin{cases} (y_1^*, A) \\ (y_2^*, A+C) \end{cases} \rightarrow \quad \begin{array}{cc} \text{What 1 gets} & \text{What 2 gets} \\ 0 & B \\ -C & B \end{array}$$

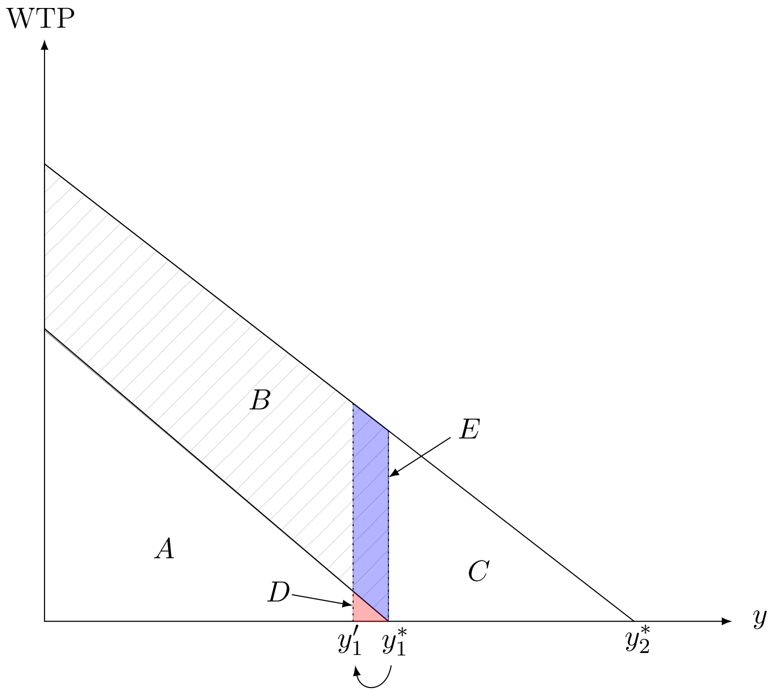
$\rightarrow$ Two bundles are self-selected, in particular $(y_2^*, A+C)$ by type 2

$\rightarrow$ Profit $= 2A + C$

- Type 2 must be given a discount of at least B or he would deviate to type 1's choice
- Due to this discount, the profit is reduced by B, compared to 1° PD
- The reduced profit B goes to the consumer as "information rent", that is the rent that must be given to an economic agent possessing "private information"

2. However, the firm can do better by slightly lowering type 1's quantity from y_1^* to y_1':

$$\text{Offer} \begin{cases} (y_1', A-D) \\ (y_2^*, A+C+E) \end{cases} \rightarrow \quad \begin{array}{cc} \text{What 1 gets} & \text{What 2 gets} \\ 0 & B-E \\ -C-E & B-E \end{array}$$



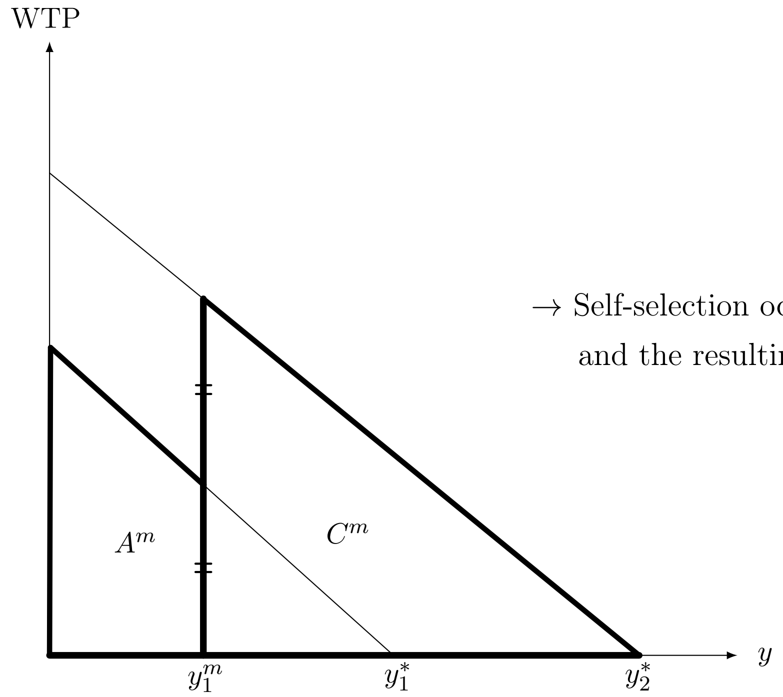
→ Two bundles are self selected, $(y_1', A - D)$ by 1 and $(y_2^*, A + C + E)$ by 2

→ Profit = $2A + C + (E - D) > 2A + C$

– Compared to the above menu, the discount (or information rent) for type 2 is reduced by E, which is a marginal gain that exceeds D, the marginal loss from type 1

3. To obtain the optimal menu, keep reducing 1's quantity until the marginal loss from 1 equals the marginal gain from 2

$$\text{Offer} \begin{cases} (y_1^m, A^m) \\ (y_2^*, A^m + C^m) \end{cases}$$



$\rightarrow$ Self-selection occurs
and the resulting profit $= 2A^m + C^m$

C. Features of the optimal menu

- Reduce the quantity of consumer with lower WTP to give less discount extract more surplus from consumer with higher WTP
- One can prove (Try this for yourself!) that $\frac{A^m}{y_1^m} > \frac{A^m+C^m}{y_2^*}$, meaning that type 2 consumer who purchases more pays less per unit, which is so called "quantity or bulk discount"

III. Third-degree price discrimination

- Suppose that the firm can tell consumers' types and thereby charge them different prices
- For some reason, however, the price has to be uniform for all units sold
- Letting $p_i(y_i), i = 1, 2$ denote the type i's (inverse) demand, the firm solves

$$\max_{y_1,y_2} p_1(y_1)y_1 + p_2(y_2)y_2 - c(y_1 + y_2)$$

$$\xrightarrow{F.O.C.} \begin{cases} \frac{d}{dy_1}\left[p_1(y_1)y_1 - c(y_1+y_2)\right] = MR_1(y_1) - MC(y_1+y_2) = 0 \\ \frac{d}{dy_2}\left[p_2(y_2)y_2 - c(y_1+y_2)\right] = MR_2(y_2) - MC(y_1+y_2) = 0 \end{cases}$$

$$\rightarrow \begin{cases} p_1(y_1)\left[1 - \frac{1}{|\varepsilon_1(y_1)|}\right] = MC(y_1+y_2) \\ p_2(y_2)\left[1 - \frac{1}{|\varepsilon_2(y_2)|}\right] = MC(y_1+y_2) \end{cases}$$

- So $p_1(y_1) < p_2(y_2)$ if and only if $|\varepsilon_1(y_1)| > |\varepsilon_2(y_2)|$ or price is higher if and only if elasticity is lower

IV. Bundling

– Suppose that there are two consumer, A and B, with the following willingness-to-pay:

Type of consumer	WTP	
	Word	Excel
A	100	60
B	60	100

→ Maximum profit from selling separately=240

→ Maximum profit from selling in a bundle=320

– Bundling is good when values for two goods are negatively correlated

V. Monopolistic competition

– Monopoly + competition: Goods that are not identical but similar

→ Downward-sloping demand curve + free entry

– The demand curve will shift in until each firm's maximized profit gets equal to 0

Ch. 26. Factor Market

I. Two faces of a firm

$$- \begin{cases} \text{Seller (supplier) in the output market with demand curve } p = p(y) \\ \text{Buyer (demander) in the factor market with supply curve } w = w(x) \end{cases}$$

$$- \text{ Factor market condition} \begin{cases} \text{One of many buyers : Competitive} \rightarrow \text{Take } w \text{ as given} \\ \text{Single buyer : Monopsonistic} \rightarrow \text{Set } w \text{ (through } x) \end{cases}$$

II. Competitive input market

A. Input choice

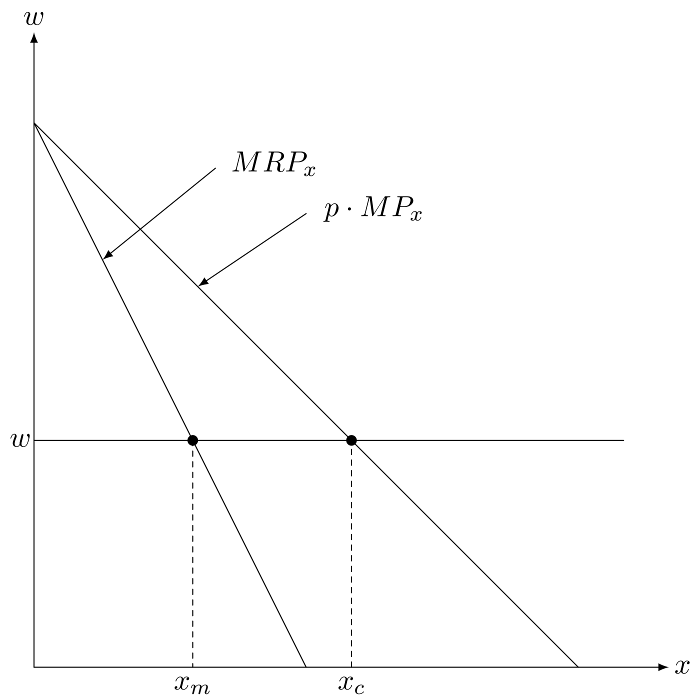
$$\max_x r(f(x)) - wx$$

$$\begin{aligned} \xrightarrow{F.O.C} r'(f(x))f'(x) = \quad & r'(y)f'(x) \quad & = \quad & w \\ & = MR \times MP & & = MC_x \\ & \equiv MRP & & \end{aligned}$$

B. Marginal revenue product

- Competitive firm in the output market: $MRP = pMP$
- Monopolist in the output market: $MRP = p\left[1 - \frac{1}{\varepsilon}\right] MP$

C. Comparison



$\rightarrow$ Monopolist buys less input than competitive firm does

III. Monopsony

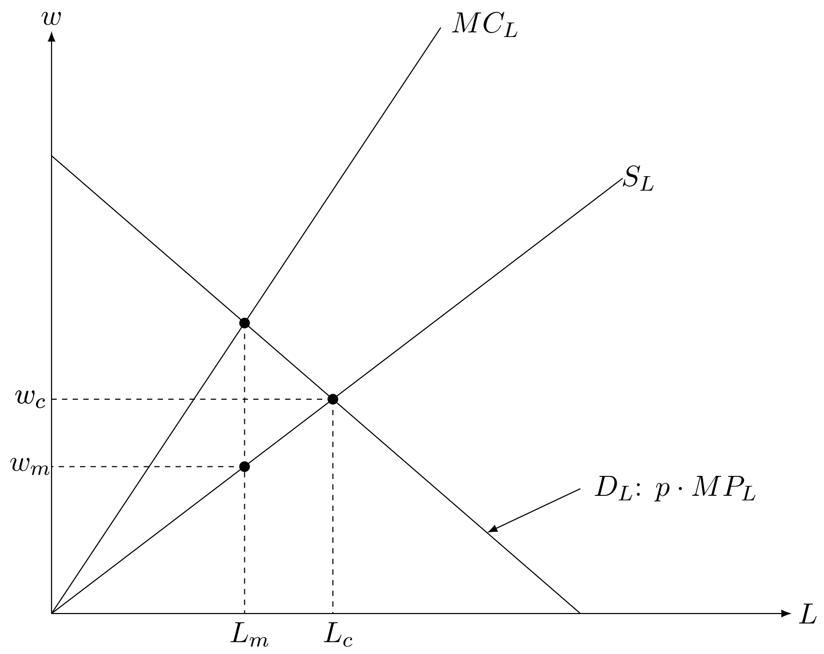
– Monopsonistic input market + Competitive output market

A. Input choice

$$\max_x \; pf(x) - w(x)x$$

$$\xrightarrow{F.O.C.} \quad \underset{= MRP}{pf'(x)} \quad \underset{= MC_x}{= w'(x)x + w(x)} = w(x)\left[1 + \frac{x}{w(x)}\frac{dw(x)}{dx}\right] = w(x)\left[1 + \frac{1}{\eta}\right],$$

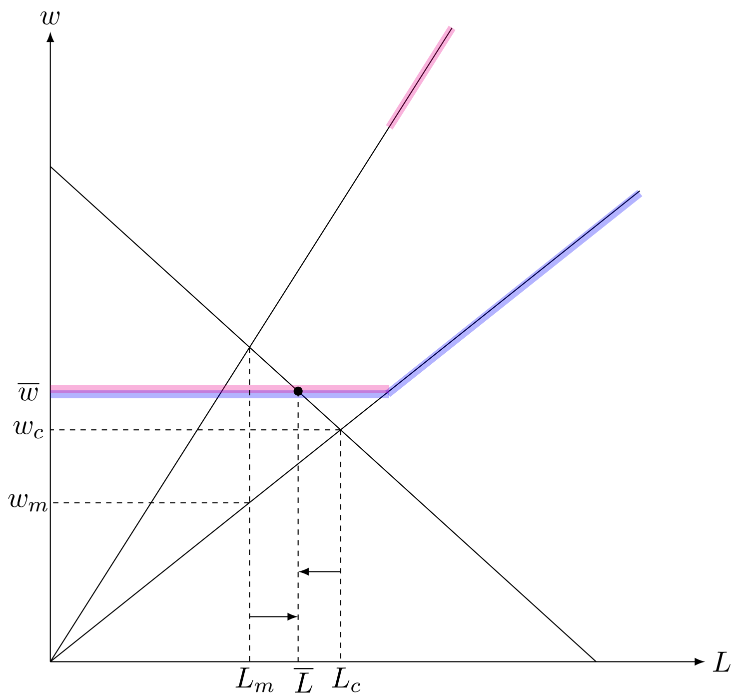
where $\eta \equiv \frac{w}{x}\frac{dx}{dw}$ or the supply elasticity of the factor



Example. $w(x) = a + bx$: (inverse) supply of factor x

$$\rightarrow MC_x = \frac{d}{dx}[w(x)x] = \frac{d}{dx}\left[ax + bx^2\right] = a + 2bx$$

B. Minimum wage under monopsony



– In competitive market, employment decreases while it increases in monopsony

IV. Upstream and downstream monopolies

– Monopolistic seller in factor market (upstream monopolist or UM)+ Monopolistic seller in output market (downstream monopolist or DM)

A. Setup

– Manufacturer (UM): Produce x at $MC = c$ and sell it at w

– Retailer (DM): Purchase x to produce y according to $y = f(x) = x$ and sell it at p

– The output demand is given as $p(y) = a - by$, $a, b > 0$

B. DM's problem:

$$\max_x \; p(f(x))f(x) - wx = (a - bx)x - wx$$

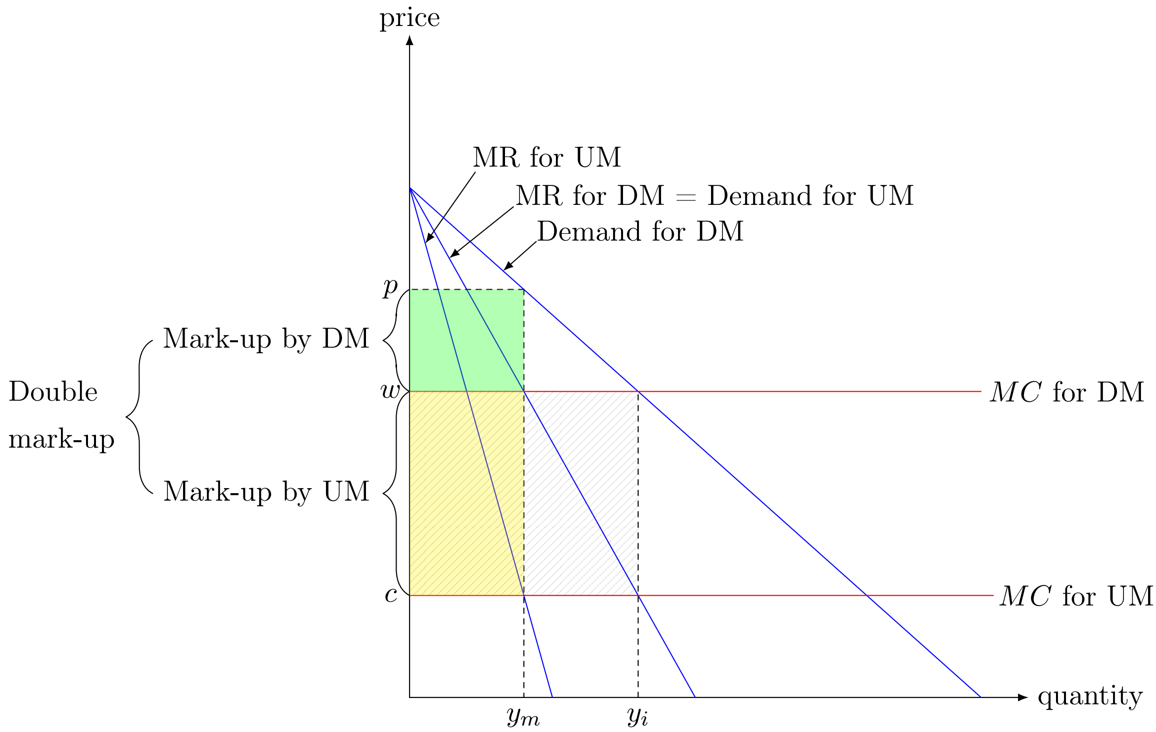
$$\xrightarrow{F.O.C.} MRP = a - 2bx = w: \text{ demand for UM}$$

C. UM's problem

$$\max_x \; (a - 2bx)x - cx$$

$$\xrightarrow{F.O.C} a - 4bx = c \;\rightarrow x = y = \frac{a-c}{4b},$$

$$\rightarrow \left\{ \begin{aligned} w &= a - 2b\frac{a-c}{4b} = \frac{a+c}{2} > c \\ p &= a - b\frac{a-c}{4b} = \frac{3a+c}{4} > \frac{a+c}{2} = w \end{aligned} \right\} \rightarrow \text{Double mark-up problem}$$



D. If 2 firms were merged, then the merged firm's problem would be

$$\max_x \; p(f(x))f(x) - cx = (a - bx)x - cx$$

$$\xrightarrow{F.O.C.} a - 2bx = c, \;\; x = y = \tfrac{a-c}{2b} > \tfrac{a-c}{4b}$$
$$p = \tfrac{a+c}{2} < \tfrac{3a+c}{4}$$

→ Integration is better in both terms of social welfare and firms' profits.

Ch. 27. Oligopoly

– Cournot model: Firms choose outputs simultaneously

– Stackelberg model: Firms choose outputs sequentially

I. Setup

– Homogeneous good produced by Firm 1 and Firm 2, y_i = Firm i's output

– Linear (inverse) demand: $p(y) = a - by = a - b(y_1 + y_2)$

– Constant MC$= c$

II. Cournot model

A. Game description

– Firm i's strategy: Choose $y_i \geq 0$

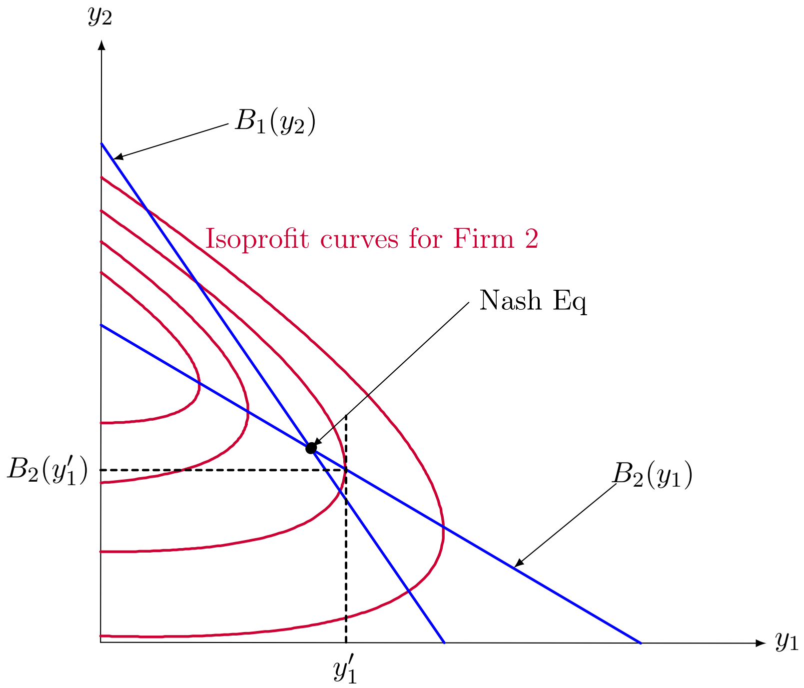
– Firm i's payoff: $\pi_i(y_1, y_2) = p(y)y_i - cy_i = (a - b(y_1 + y_2) - c)y_i$

B. Best response (BR): To calculate Firm 1's BR to Firm 2's strategy y_2, solve

$$\max_{y_1 \geq 0} \pi_1(y_1, y_2) = (a - b(y_1 + y_2) - c)y_1$$

$$\xrightarrow{F.O.C.} \frac{d}{dy_1}\pi_1(y_1, y_2) = a - b(y_1 + y_2) - c - by_1 = 0 \rightarrow \ 2by_1 = a - c - by_2$$

$$\rightarrow \begin{cases} B_1(y_2) = \frac{a-c-by_2}{2b} \\ B_2(y_1) = \frac{a-c-by_1}{2b} \end{cases}$$



C. Nash equilibrium: Letting y_i^c denote i' equilibrium quantity,

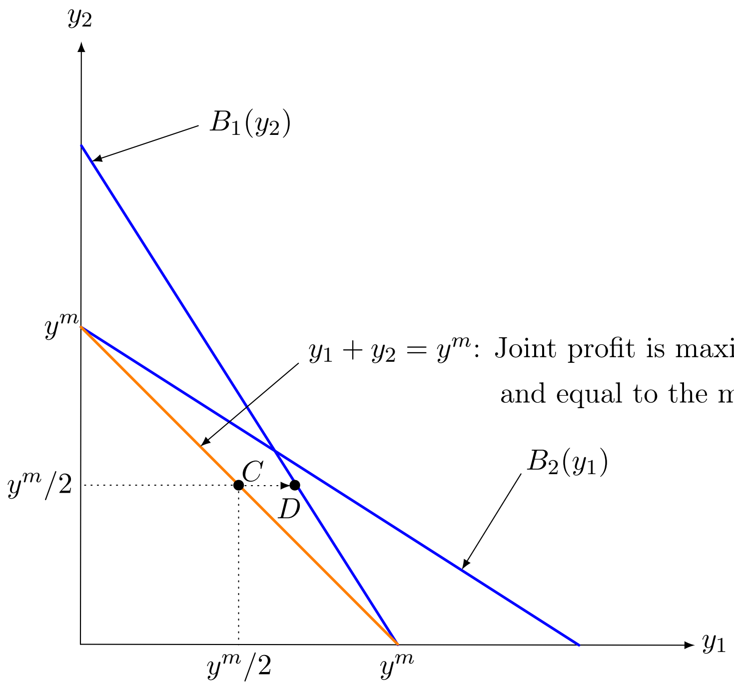
$$\rightarrow \begin{cases} B_1(y_2^c) = y_1^c \\ B_2(y_1^c) = y_2^c \end{cases} \rightarrow \begin{cases} y_1^c = y_2^c = y^c \equiv \frac{a-c}{3b} \\ \pi^c = (a - 2by^c - c)y^c = \frac{(a-c)^2}{9b} \end{cases}$$

D. Comparison with monopoly

- Monopolist's solution:

$$\left.\begin{aligned} y^m &= \tfrac{a-c}{2b} \\ \pi^m &= \tfrac{(a-c)^2}{4b} \end{aligned}\right\} \rightarrow \begin{cases} y^m < 2y^c \\ \pi^m > 2\pi^c \end{cases}$$

- Collusion with each firm producing $\frac{y^m}{2}$ is not sustainable



E. Oligopoly with n firms

- Letting $y \equiv \sum_{i=1}^{n} y_i$, Firm i solves

$$\max_{y_i} \ (p(y) - c)y_i = (a - b(y_1 + \cdots + y_i + \cdots + y_n) - c)\, y_i$$

$$\xrightarrow{F.O.C} \ -by_i + (a - by - c) = 0.$$

- Since firms are symmetric, we have $y_1 = y_2 = \cdots = y_n = y^c$, with which the F.O.C becomes

$$-by^c + (a - nby^c - c) = 0$$

$$\rightarrow y^c = \frac{a-c}{(n+1)b}, \ y = ny^c = \frac{n}{n+1}\frac{a-c}{b} \rightarrow \frac{a-c}{b} \text{ as } n \rightarrow \infty$$

- Note that $\frac{a-c}{b}$ is the competitive quantity. So the total quantity increases toward the competitive level as there are more and more firms in the market.

III. Stackelberg model

A. Game description

– Firm 1 first chooses y_1, which Firm 2 observes and then chooses y_2

$\rightarrow$ Firm 1: (Stackelberg) leader, Firm 2: follower

– Strategy

1) Firm 1: y_1

2) Firm 2: $r(y_1)$, a function or plan contingent on what Firm 1 has chosen

B. Subgame perfect equilibrium

– Backward induction: Solve first the profit maximization problem of Firm 2

1) Given Firm 1's choice y_1, Firm 2 chooses $y_2 = r(y_1)$ to solve

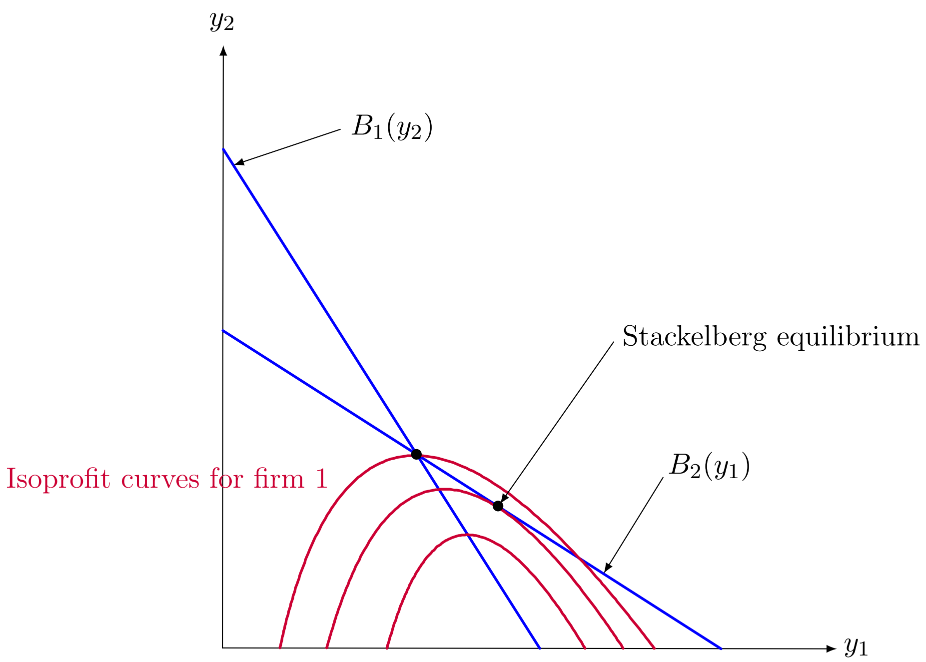
$$\max_{y_2} \ \pi_2(y_1, y_2)$$

$\xrightarrow{F.O.C.} r(y_1) = \frac{a-c-by_1}{2b}$, which is the same as $B_2(y_1)$.

2) Understanding that Firm 2 will respond to y_1 with quantity $r(y_1)$, Firm 1 will choose y_1 to solve

$$\max_{y_1} \pi_1(y_1, r(y_1)) = (a - b(y_1 + r(y_1)) - c)y_1 = \frac{1}{2}(a - c - by_1)y_1$$

$$\xrightarrow{F.O.C} (a - c - by_1) - by_1 = 0 \rightarrow y_1^s = \frac{a-c}{2b}, y_2^s = r(y_1^s) = \frac{a-c}{4b} < y_1^s$$



IV. Bertrand model

A. Game description

– Two firms compete with each other using price instead quantity

– Whoever sets a lower price takes the entire market (equally divide the market if prices are equal)

B. Nash equilibrium: $p_1 = p_2 = c$ is a unique NE where both firm split the market and get zero profit, whose proof consists of 3 steps:

1. $p_1 = p_2 = c$ *constitutes a Nash equilibrium*
   – If, for instance, Firm 1 deviates to $p_1 > c$, it continues to earn zero profit
   – If Firm 1 deviates to $p_1 < c$, it incurs a loss
   – Thus, $p_1 = c$ is a Firm 1's best response to $p_2 = c$
2. *There is no Nash equilibrium where* $\min\{p_1, p_2\} < c$
   – At least one firm incurs a loss
3. *There is no Nash equilibrium where* $\max\{p_1, p_2\} > c$
   – $p_1 = p_2 > c$: Firm 1, for instance, would like to slightly lower the price to take the entire market rather than a half, though the margin gets slightly smaller
   – $p_1 > p_2 > c$: Firm 1 would like to cut its price slightly below p_2 to take the entire market and enjoy some positive, instead zero, profit

Ch. 28. Game Theory

– Studies how people behave in a strategic situation where one's payoff depends on others' actions as well as his

I. Strategic situation: Players, strategies, and payoffs

A. Example: 'Prisoner's dilemma' (PD)

– Kim and Chung: suspects for a bank robbery

– If both confess, '3 months in prison' for each

– If only one confesses, 'go free' for him and '6 months in prison' for the other

– If both denies, '1 months in prison' for each

		Chung	
		Confess	Deny
Kim	Confess	-3, $\boxed{-3}$	0, -6
	Deny	-6, $\boxed{0}$	-1, -1

B. Dominant st. equilibrium (DE)

– A strategy of a player is *dominant* if it is optimal for him no matter what others are doing

– A strategy combination is *DE* if each player's strategy is dominant

– In PD, 'Confess' is a dominant strategy → (Confess,Confess) is DE

– (Deny,Deny) is mutually beneficial but not sustainable

C. Example with No DE: Capacity expansion game

		Sony		
		Build Large	Build Small	Do not Build
Samsung	Build Large	0, 0	12, 8	18, $\boxed{9}$
	Build Small	0, 12	16, $\boxed{16}$	20, 15
	Do not Build	9, 18	15, $\boxed{20}$	18, 18

– No dominant strategy for either player

– However, (Build Small, Build Small) is a reasonable prediction

D. Nash equilibrium (NE)

- A strategy combination is NE if each player's strategy is optimal given others' equilibrium strategies
- In the game of capacity expansion, (Build Small, Build Small) is a NE
- Example with multiple NE: Battle of sexes game

		Sheila	
		K1	Soap Opera
Bob	K1	2, $\boxed{1}$	0, 0
	Soap Opera	0, 0	1, $\boxed{2}$

→ NE: (K1, K1) and (Soap opera, Soap opera)

E. Location game

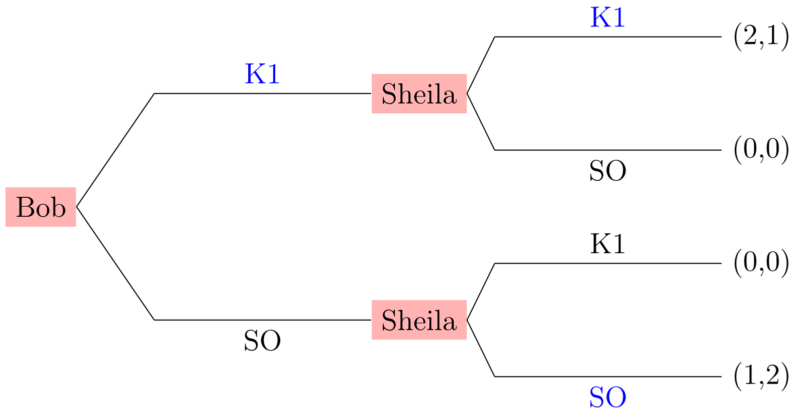
1. Setup
   - Bob and Sheila: 2 vendors on the beach $[0, 1]$
   - Consumers are evenly distributed along the beach
   - With price being identical and fixed, vendors choose locations
   - Each consumer prefers a shorter walking distance
2. Unique NE: (1/2,1/2)
3. Socially optimal locations: (1/4,3/4)
4. Applications: Product differentiation, majority voting and median voter
5. NE with 3 vendors → no NE!

II. Sequential games

A. Example: A *sequential* version of battle of sexes

- Bob moves first to choose between 'K1' and 'Soap opera'
- Observing Bob's choice, Sheila chooses her strategy

B. Game tree



C. Strategies $\begin{cases} \text{Bob's strategy: K1, SO} \\ \text{Sheila's strategy: K1} \cdot \text{K1, K1} \cdot \text{SO, SO} \cdot \text{K1, SO} \cdot \text{SO} \end{cases}$

		Sheila							
		K1·K1		K1·SO		SO·K1		SO·SO	
Bob	K1	2	$\boxed{1}$	2	$\boxed{1}$	0	0	0	0
	SO	0	0	1	$\boxed{2}$	0	0	1	$\boxed{2}$

→ All NE: $(\text{K1}, \text{K1} \cdot \text{K1})$, $(\text{SO}, \ \text{SO} \cdot \text{SO})$, and $(\text{K1}, \text{K1} \cdot \text{SO})$

D. All other NE than $(\text{K1}, \text{K1} \cdot \text{SO})$ are problematic

– For instance, $(\text{SO}, \text{SO} \cdot \text{SO})$ involves an incredible threat

– In both $(\text{SO}, \text{SO} \cdot \text{SO})$and $(\text{K1}, \text{K1} \cdot \text{K1})$, Sheila is not choosing optimally when Bob (unexpectedly) chooses a non-equilibrium strategy.

E. Subgame perfect equilibrium (SPE)

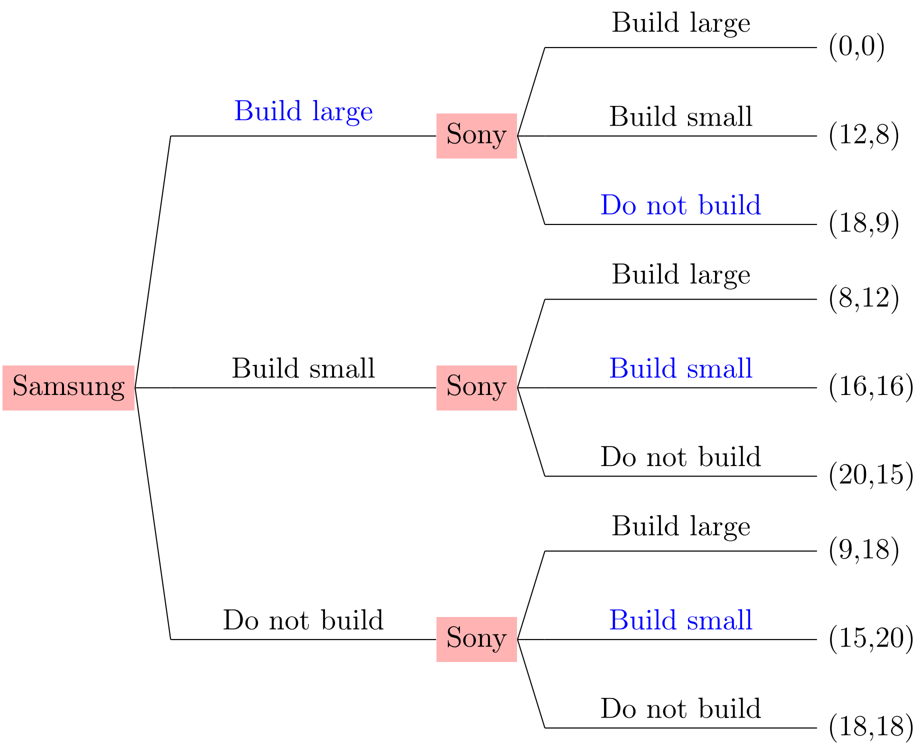
– Requires that each player chooses optimally whenever it is his/her turn to move.

– Often, not all NE are SPE: $(\text{K1}, \text{K1} \cdot \text{SO})$ is the only SPE in the above game.

– SPE is also called a backward induction equilibrium.

– In sum, not all NE is an SPE while SPE must always be an NE.

F. Another example: Modify the capacity expansion game to let Samsung move first



- Unique SPE strategy: Sony chooses $\left\{ \begin{array}{ll} \text{Do not build} & \text{if Samsung chooses Build large} \\ \text{Build small} & \text{otherwise} \end{array} \right\}$

  while Samsung chooses Build large

  $\rightarrow$ SPE outcome: (Build large,Do not build)

- There exists NE that is not SPE: For instance, Sony chooses 'always Build small' while Samsung chooses 'Build small'

III. Repeated games: A sequential game where players repeatedly face the same strategic situation

- Can explain why people can cooperate in games like prisoner's dilemma

A. Infinite repetition of PD

- Play the same PD every period r

- Tomorrow's payoff is discounted by discount rate$= \delta < 1$

  $\rightarrow$ higher δ means that future payoffs are more important

B. Equilibrium strategies sustaining cooperation:

(i) Grim trigger strategy: I will deny as long as you deny while I will confess forever once you confess

$$\text{Deny today}: \; -1+\delta(-1)+\delta^2(-1)+\cdots = \frac{-1}{1-\delta}$$
$$\text{Confess today}: \; 0+\delta(-3)+\delta^2(-3)+\cdots = \frac{-3\delta}{1-\delta}$$

So if $\delta > \frac{1}{3}$, 'Confess' is better

(ii) Tit-for-tat strategy: I will deny (confess) tomorrow if you deny (confess) today $\rightarrow$ Most popular in the lab. experiment by Axerlod

C. Application: enforcing a cartel (airline pricing)

D. Finitely or infinitely repeated?

Ch. 30. Exchange

– Partial equilibrium analysis: Study how price and output are determined in a *single* market, taking as given the prices in all other markets.

– General equilibrium analysis: Study how price and output are *simultaneously* determined in *all* markets.

I. Exchange Economy

A. Description of the economy

– Two goods, 1 and 2, and two consumers, A and B.

– Initial endowment allocation: (ω_i^1, ω_i^2) for consumer $i = A$ or B.

– Allocation: (x_i^1, x_i^2) for consumer $i = A$ or B.

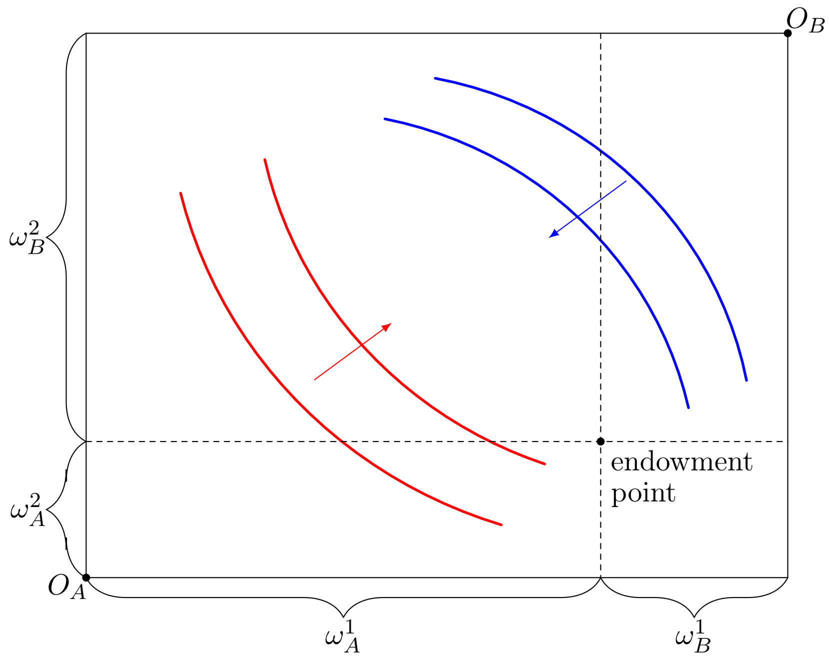
– Utility: $u_i(x_i^1, x_i^2)$ for consumer $i = A$ or B.

B. Edgeworth box

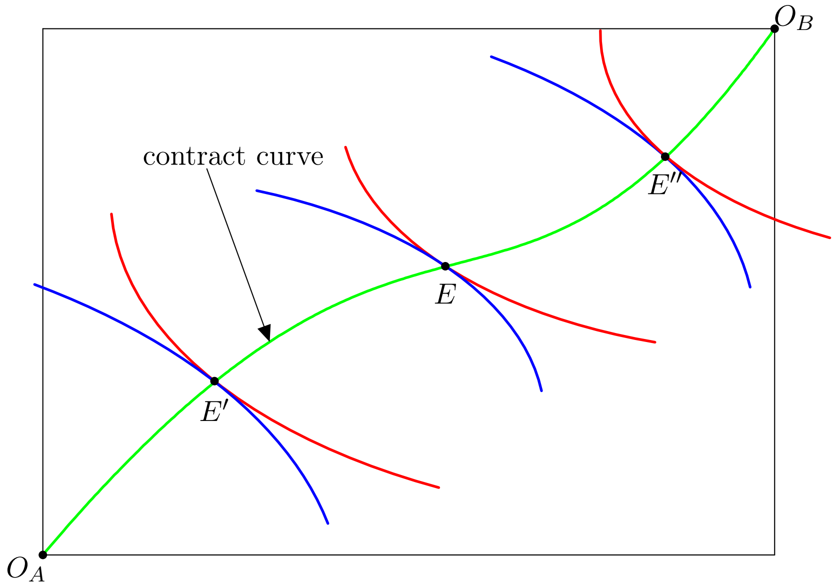
– Allocation is called *feasible* if the total consumption is equal to the total endowment:

$$x_A^1 + x_B^1 = \omega_A^1 + \omega_B^1$$
$$x_A^2 + x_B^2 = \omega_A^2 + \omega_B^2.$$

- All feasible allocations can be illustrated using *Edgeworth box*



– An allocation is *Pareto efficient* if there is no other allocation that makes both consumers better off



$\rightarrow$ The set of Pareto efficient points is called the *contract curve.*

II. Trade and Market Equilibrium

A. Utility maximization

– Given the market prices (p_1, p_2), each consumer i solves

$$\max_{(x_i^1, x_i^2)} \; u_i(x_i^1, x_i^2) \quad \text{subject to} \;\; p_1 x_i^1 + p_2 x_i^2 = p_1 \omega_i^1 + p_2 \omega_i^2 \equiv m_i(p_1, p_2),$$

which yields the demand functions $(x_i^1(p_1, p_2, m_i(p_1, p_2)), x_i^2(p_1, p_2, m_i(p_1, p_2)))$.

B. Excess demand function and equilibrium prices

– Define the net demand function for each consumer i and each good k as

$$e_i^k(p_1, p_2) \equiv x_i^k(p_1, p_2, m_i(p_1, p_2)) - \omega_i^k.$$

– Define the aggregate excess demand function for each good k as

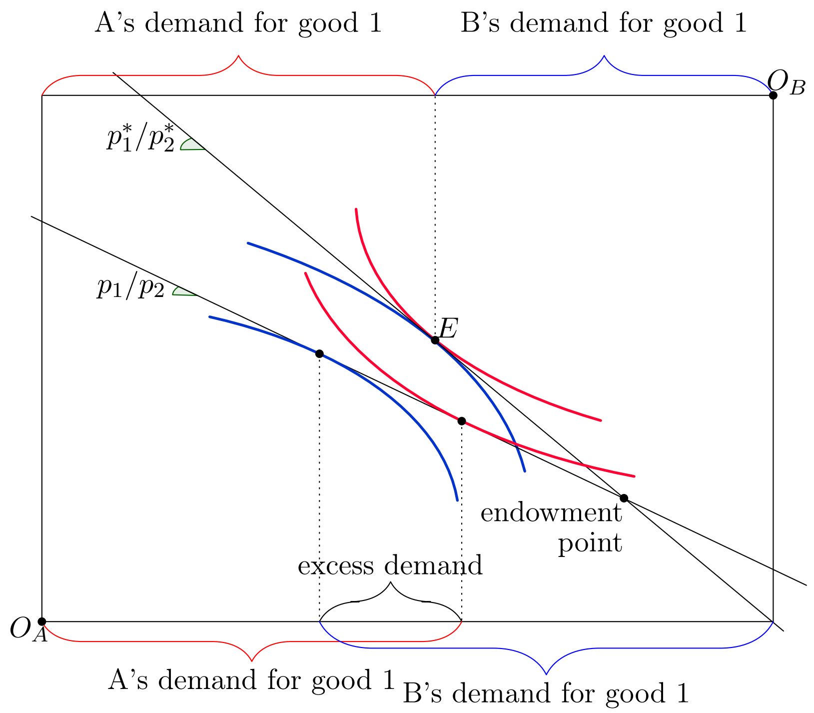
$$\begin{aligned} z_k(p_1, p_2) &\equiv e_A^k(p_1, p_2) + e_B^k(p_1, p_2) \\ &= x_A^k(p_1, p_2, m_A(p_1, p_2)) + x_B^k(p_1, p_2, m_B(p_1, p_2)) - \omega_A^k - \omega_B^k, \end{aligned}$$

i.e. the amount by which the total demand for good k exceeds the total supply.

– If $z_k(p_1, p_2) > (<) 0$, then we say that good k is in *excess demand* (*excess supply*).

– At the equilibrium prices (p_1^*, p_2^*), we must have neither excess demand nor excess supply, that is

$$z_k(p_1^*, p_2^*) = 0, \;\; k = 1, 2.$$



– If (p_1^*, p_2^*) is equilibrium prices, then (tp_1^*, tp_2^*) for any $t > 0$ is equilibrium prices as well so only the relative prices p_1^*/p_2^* can be determined.

– A technical tip: Set $p_2 = 1$ and ask what p_1 must be equal to in equilibrium.

C. Walras' Law

– The value of aggregate excess demand is identically zero, i.e.

$$p_1 z_1(p_1, p_2) + p_2 z_2(p_1, p_2) \equiv 0.$$

– The proof simply follows from adding up two consumers' budget constraints

$$\begin{array}{rl} & p_1 e_A^1(p_1,p_2) + p_2 e_A^2(p_1,p_2) = 0 \\ + & p_1 e_B^1(p_1,p_2) + p_2 e_B^2(p_1,p_2) = 0 \\ \hline & p_1[\underbrace{e_A^1(p_1,p_2) + e_B^1(p_1,p_2)}_{z_1(p_1,p_2)}] + p_2[\underbrace{e_A^2(p_1,p_2) + e_B^2(p_1,p_2)}_{z_2(p_1,p_2)}] = 0 \end{array}$$

– Any prices (p_1^*, p_2^*) that make the demand and supply equal in one market, is guaranteed to do the same in the other market

– Implication: Need to find the prices (p_1^*, p_2^*) that clear one market only, say market 1,

$$z_1(p_1^*, p_2^*) = 0.$$

– In general, if there are markets for n goods, then we only need to find a set of prices that clear $n - 1$ markets.

D. Example: $u_A(x_A^1, x_A^2) = (x_A^1)^a (x_A^2)^{1-a}$ and $u_B(x_B^1, x_B^2) = (x_B^1)^b (x_B^2)^{1-b}$

– From the utility maximization,

$$x_A^1(p_1, p_2, m) = a\frac{m_A(p_1,p_2)}{p_1} = a\frac{p_1\omega_A^1 + p_2\omega_A^2}{p_1}$$
$$x_B^1(p_1, p_2, m) = b\frac{m_B(p_1,p_2)}{p_1} = b\frac{p_1\omega_B^1 + p_2\omega_B^2}{p_1}$$

– So,

$$z_1(p_1, 1) = a\frac{p_1\omega_A^1 + \omega_A^2}{p_1} + b\frac{p_1\omega_B^1 + \omega_B^2}{p_1} - \omega_A^1 - \omega_B^1.$$

– Setting $z_1(p_1, 1) = 0$ yields

$$p_1^* = \frac{a\omega_A^2 + b\omega_B^2}{(1-a)\omega_A^1 + (1-b)\omega_B^1}.$$

III. Equilibrium and Efficiency: The First Theorem of Welfare Economics

– First Theorem of Welfare Economics: Any eq. allocation in the competitive market must be Pareto efficient.

– While this result should be immediate from a graphical illustration, a more formal proof is as follows:

(i) Suppose that an eq. allocation $(x_A^1, x_A^2, x_B^1, x_B^2)$ under eq. prices (p_1^*, p_2^*) is not Pareto efficient, which means there is an alternative allocation $(y_A^1, y_A^2, y_B^1, y_B^2)$ that is feasible

$$\begin{aligned} y_A^1 + y_B^1 &= \omega_A^1 + \omega_B^1 \\ y_A^2 + y_B^2 &= \omega_A^2 + \omega_B^2 \end{aligned} \tag{4}$$

and makes both consumers better off

$$\begin{aligned} (y_A^1, y_A^2) &\succ_A (x_A^1, x_A^2) \\ (y_B^1, y_B^2) &\succ_B (x_B^1, x_B^2). \end{aligned}$$

(ii) The fact that (x_A^1, x_A^2) and (x_B^1, x_B^2) solve the utility maximization problem implies

$$\begin{aligned} p_1^* y_A^1 + p_2^* y_A^2 &> p_1^*\omega_A^1 + p_2^*\omega_A^2 \\ p_1^* y_B^1 + p_2^* y_B^2 &> p_1^*\omega_B^1 + p_2^*\omega_B^2. \end{aligned} \tag{5}$$

(iii) Sum up the inequalities in (5) side by side to obtain

$$p_1^*(y_A^1 + y_B^1) + p_2^*(y_A^2 + y_B^2) > p_1^*(\omega_A^1 + \omega_B^1) + p_2^*(\omega_A^2 + \omega_B^2),$$

which contradicts with equations in (4).

– According to this theorem, the competitive market is an excellent economic mechanism to achieve the Pareto efficient outcomes.

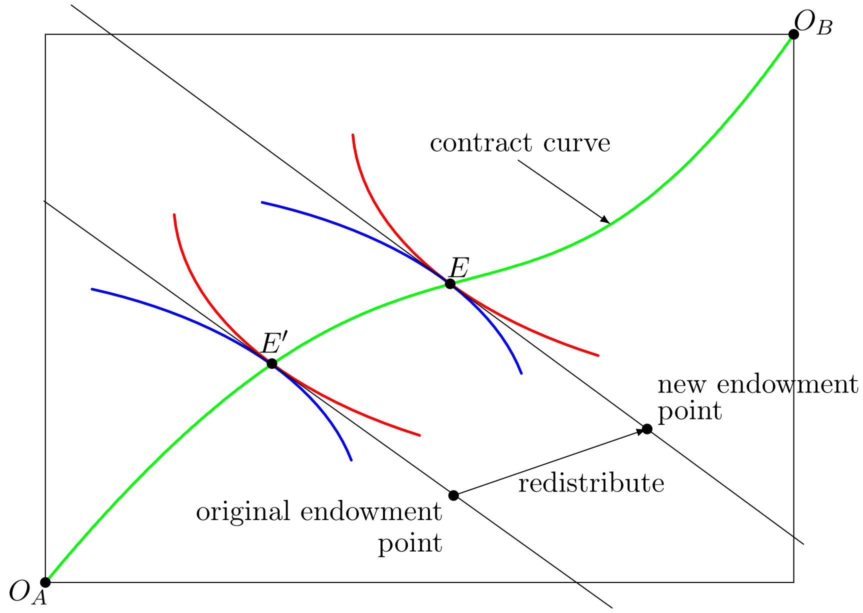
– Limitations:

(i) Competitive behavior

(ii) Existence of a market for every possible good (even for externalities)

IV. Efficiency and Equilibrium: The Second Theorem of Welfare Economics

– Second Theorem of Welfare Economics: If all consumers have convex preferences, then there will always be a set of prices such that each Pareto efficient allocation is a market eq. for an appropriate assignment of endowments.



– The assignment of endowments can be done using some non-distortionary tax.

– Implication: Whatever welfare criterion we adopt, we can use competitive markets to achieve it

– Limitations:

(i) Competitive behavior

(ii) Hard to find a non-distortionary tax

(iii) Lack of information and enforcement power